

# On Model-Checking Concurrent Recursive Programs

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# Concurrent Software Model-Checking

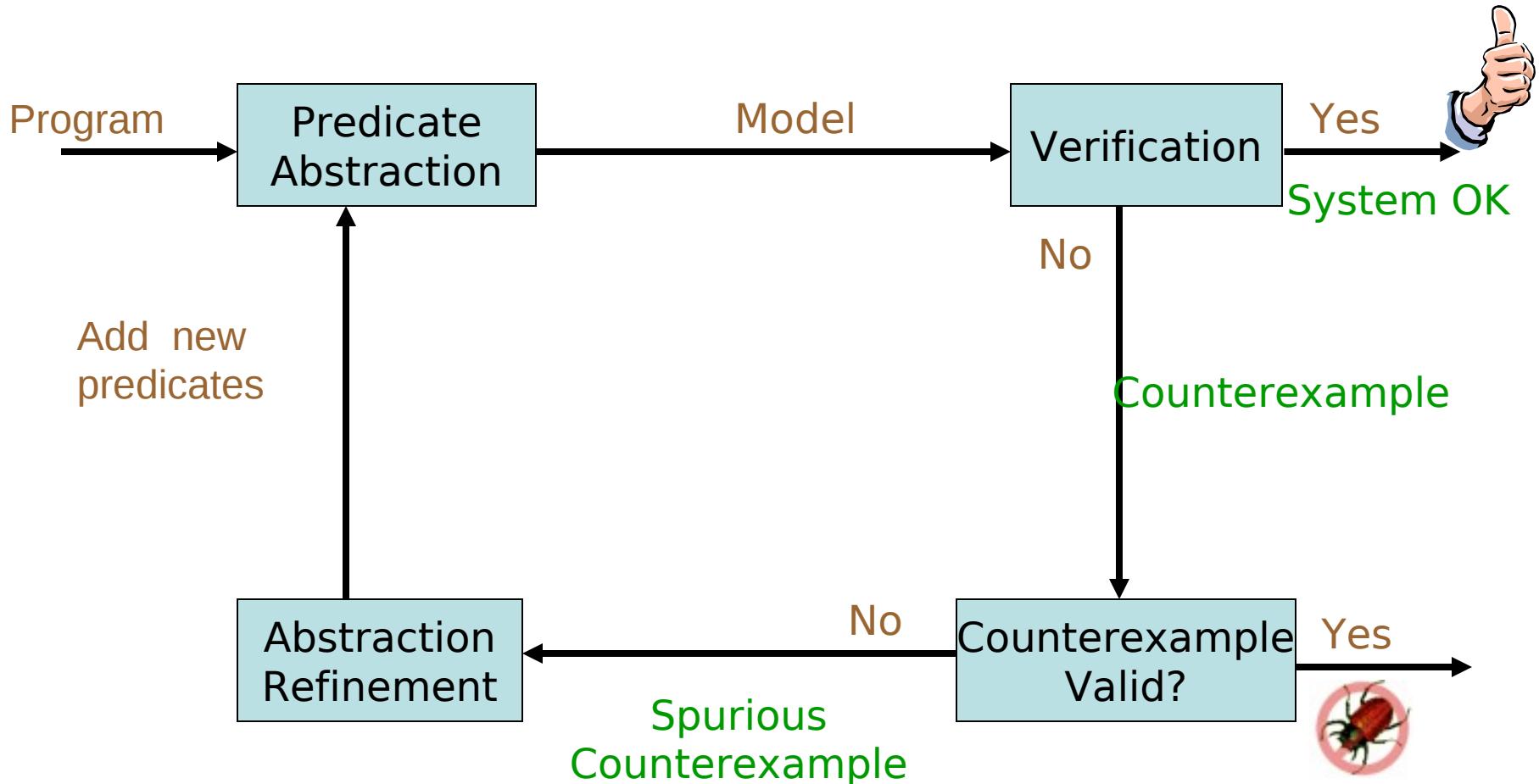
- A major challenge in the community
- Various complex features:
  - Data over unbounded (very large) domains
  - Presence of recursive procedure calls
  - Concurrency and synchronisation
  - Dynamism
- Reachability of a control point is undecidable [Ramalingam 2000]
- Any analysis technique is incomplete



# Existing Works

- Large data: Predicate abstraction [Graf, Saidi'97]
- $x:\text{int}; \quad (x>5) \text{ and } (x\leq 5)$
- Model precision and complexity of the analysis:  
number of predicates
- Discover a small number of predicates needed to prove the property
- Counter Example Guided Abstraction Refinement (CEGAR)

# CEGAR



# Existing Works: CEGAR

- Implemented in several tools:
- SLAM : no concurrency
- MAGIC: no recursion
- BLAST: no recursion+concurrency
- etc

# Existing Works (recursion+concurrency)

- Pushdown systems: Sequential recursive programs [Esparza, Knoop'99], [Esparza, Schwoon'01]
- Model Recursion + Represent infinite configurations of recursive programs by regular languages [Bouajjani, Esparza, Maler'97],
- Tool: MOPED
- Compositions of PDSs: Concurrent recursive programs [Bouajjani, Esparza, T., Qadeer, Rehof,.....]



**Data assumed to have a small domain**

# This Work

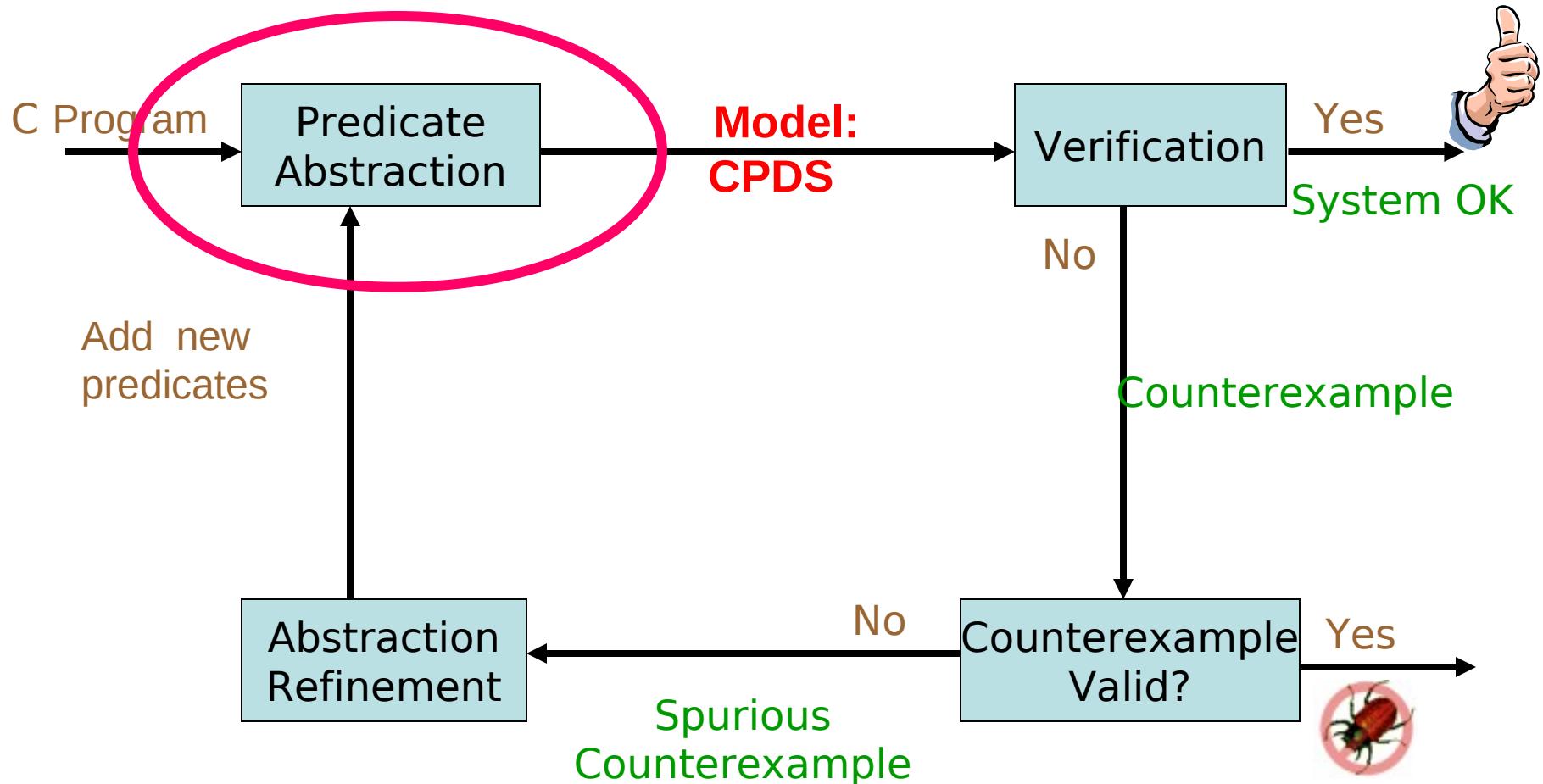
- Handle
  - Large data
  - Recursion
  - Concurrency



# Our Approach

- Large Data: CEGAR on predicate abstraction
- Concurrency and recursion:  
Communicating PDSs

# Our Approach



# Pushdown System: Definition

Pushdown System :  $S = (Q, Act, \Gamma, c_0, \Delta)$ :

$Q$  is a finite set of states

$Act$  is a finite set of actions

$\Gamma$  is a finite stack alphabet

$c_0$  is the initial configuration  $(q, v), q \in Q, v \in \Gamma^*$

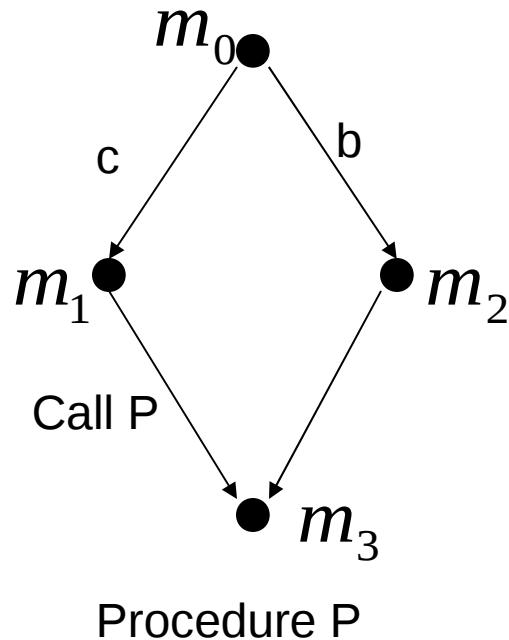
$\Delta$  is a finite set of rules of the form

$(q, \gamma) \xrightarrow{a} (q', w), q, q' \in Q, a \in Act, \gamma \in \Gamma, w \in \Gamma^*$

Transition relation:  $(q, \gamma u) \xrightarrow{a} (q', wu)$

# From a Sequential Program to a Pushdown System

[Esparza,Schwoon'01]



- $$r_1 : (glob, (m_0, loc)) \xrightarrow{c} (glob', (m_1, loc'))$$
- $$r_2 : (glob, (m_0, loc)) \xrightarrow{b} (glob', (m_2, loc'))$$
- $$r_3 : (glob, (m_1, loc)) \xrightarrow{\tau} (glob', (m_0, loc'))(m_3, loc)$$
- $$r_4 : (glob, (m_2, loc)) \xrightarrow{\tau} (glob, (m_3, loc))$$
- $$r_5 : (glob, (m_3, loc)) \xrightarrow{\tau} (glob, \varepsilon)$$

- States: Global variables
  - Symbols of the stack: Local variables+control points
- Predicates?**

# Predicates?

- Subset  $C$  of the conditions of the program
- Close it by computing weakest preconditions w.r.t. statements of the program

$$s : x := e$$
$$WP_s(p) : p[x \leftarrow e]$$

# Predicates?

- Subset  $C$  of the conditions of the program
- Close it by computing weakest preconditions w.r.t. statements of the program

Initially  $C$  empty

Conditions are added using CEGAR

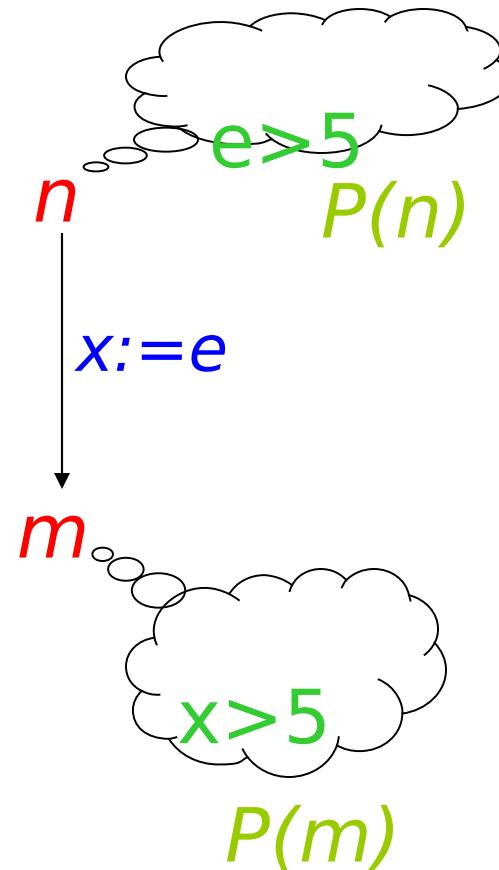
# Compute the predicates?

- Associate to each control point  $n$  a set of predicates  $P(n) = P(n)_{loc} + P(n)_{glob}$
- $P(n)$  : set of predicates needed at point  $n$
- Initially,  $P(n) = \emptyset$  for all  $n$
- $P(n)$  : updated by computing weakest preconditions

# Compute $P(n)$

$s: n \rightarrow m$

- $s$ : assignment; add  $WP_s(P(m))$  to  $P(n)$



# Compute $P(n)$

$s: n \rightarrow m$

**S:** goto or synchronisation statement:

add  $P(m)$  to  $P(n)$

**S:** (if  $c$  then)      add  $P(m)$  to  $P(n)$

$c$  in  $C$  : add  $c$  to  $P(n)$

**S:** call to a procedure  $q$

add  $P(m)_{loc}$  and  $P(init-q)_{glob}$  to  $P(n)$

How to compute the  
PushDown System?

# The PDS rules

$s: n \rightarrow m : \text{goto}$

$(\text{glob}, (n, \text{loc})) \rightarrow (\text{glob}', (m, \text{loc}'))$

$\text{loc} \in P(n)_{\text{loc}}$

$\text{loc}' \in P(m)_{\text{loc}}$

$\text{glob} \in P(n)_{\text{glob}}$

$\text{glob}' \in P(m)_{\text{glob}}$

$(\text{loc} \wedge \text{loc}')$  satisfiable

$(\text{glob} \wedge \text{glob}')$  satisfiable

**Undecidable** for first  
order formulas over the  
integers

**SIMPLIFY:** sound theorem prover that answers  
true, false, or unknown

# The PDS rules

$s: n \rightarrow m : \text{assignment}$

$(glob, (n, loc)) \rightarrow (glob', (m, loc'))$

$loc \in P(n)_{loc} \quad loc' \in P(m)_{loc}$

$glob \in P(n)_{glob} \quad glob' \in P(m)_{glob}$

$(loc \wedge WP_s(loc')) \quad \text{satisfiable}$

$(glob \wedge WP_s(glob')) \quad \text{satisfiable}$

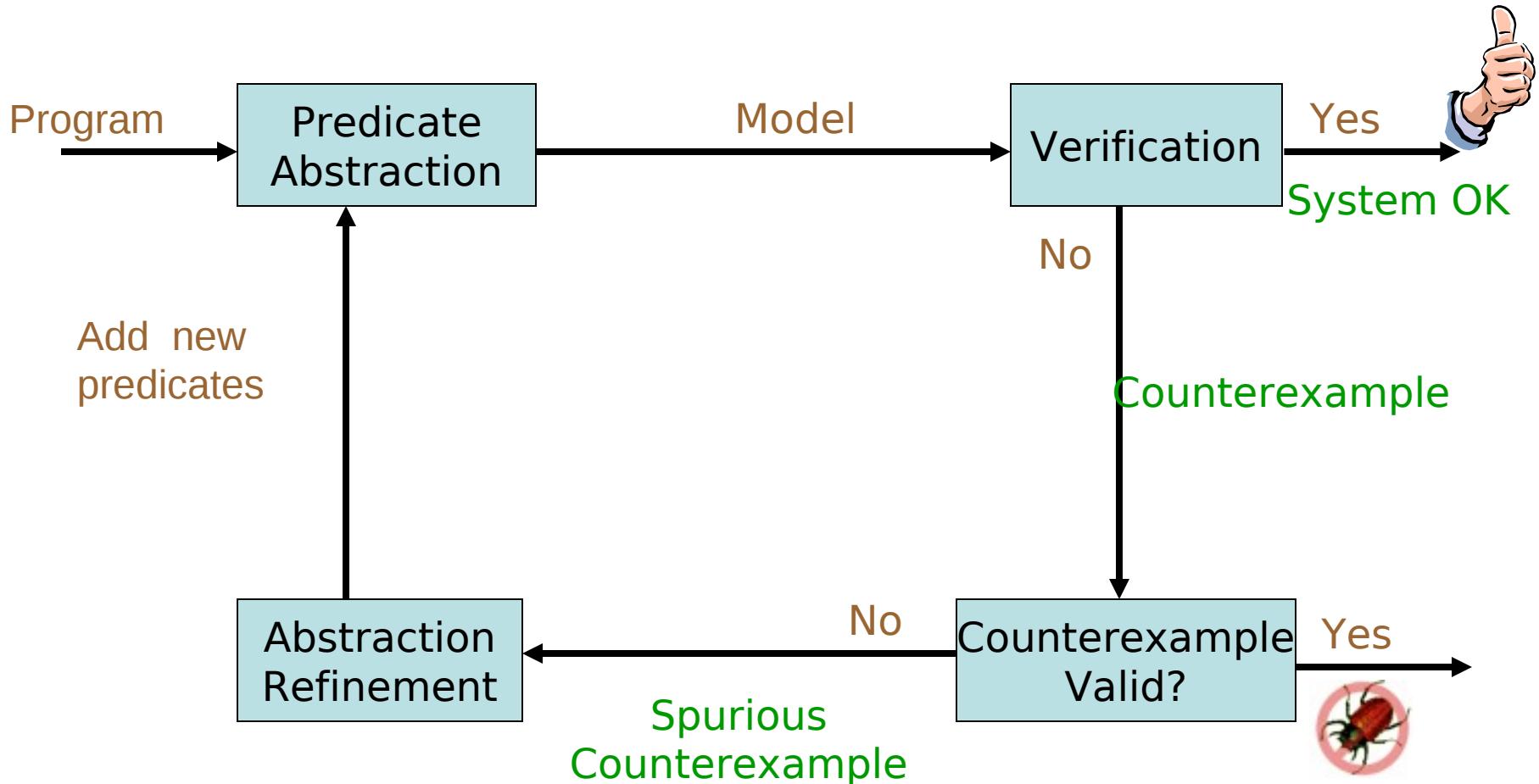
# Predicates?

- Subset  $C$  of the conditions of the program
- Close it by computing weakest preconditions w.r.t. statements of the program

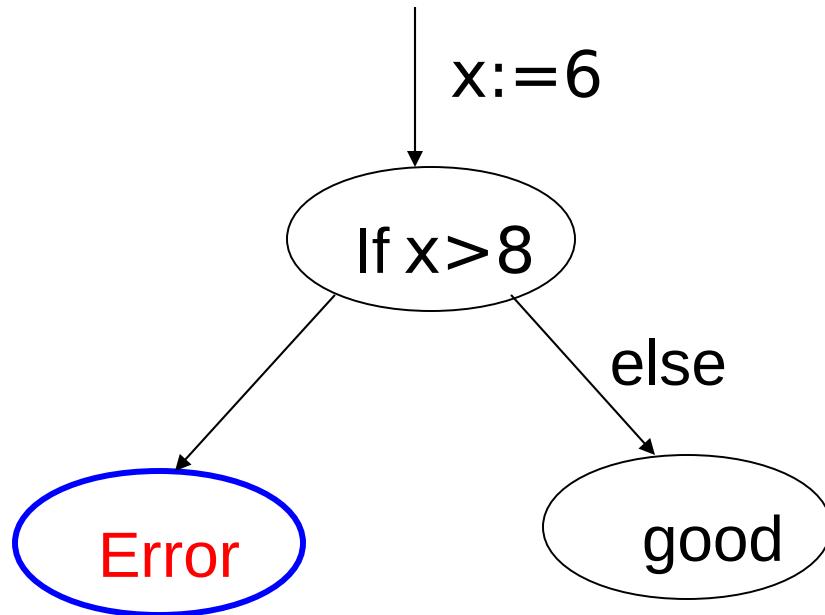
Initially  $C$  empty

Conditions are added using CEGAR

# CEGAR



# Example

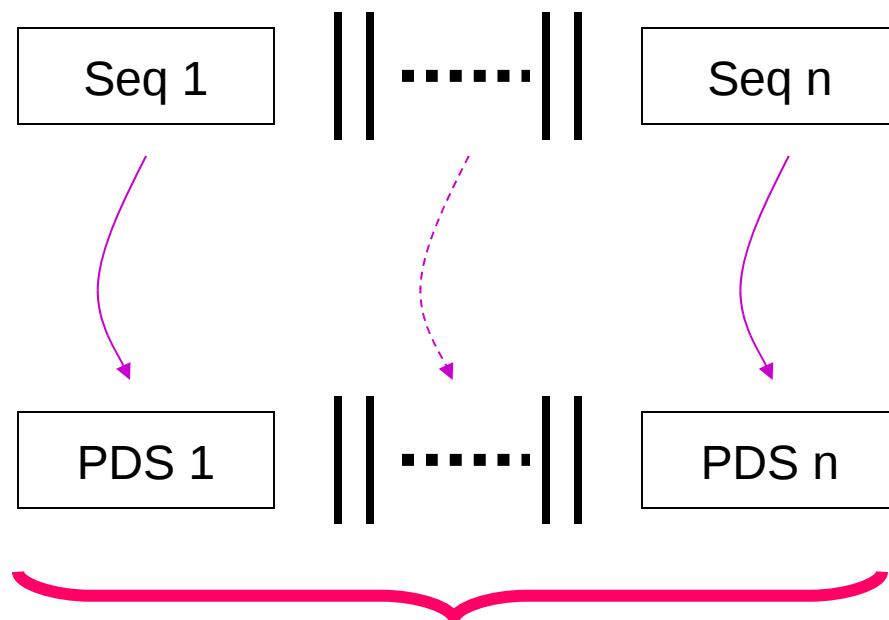


$C$  is empty: Error reachable

$(x > 8)$  in  $C$  : Error non reachable

# What about Concurrency?

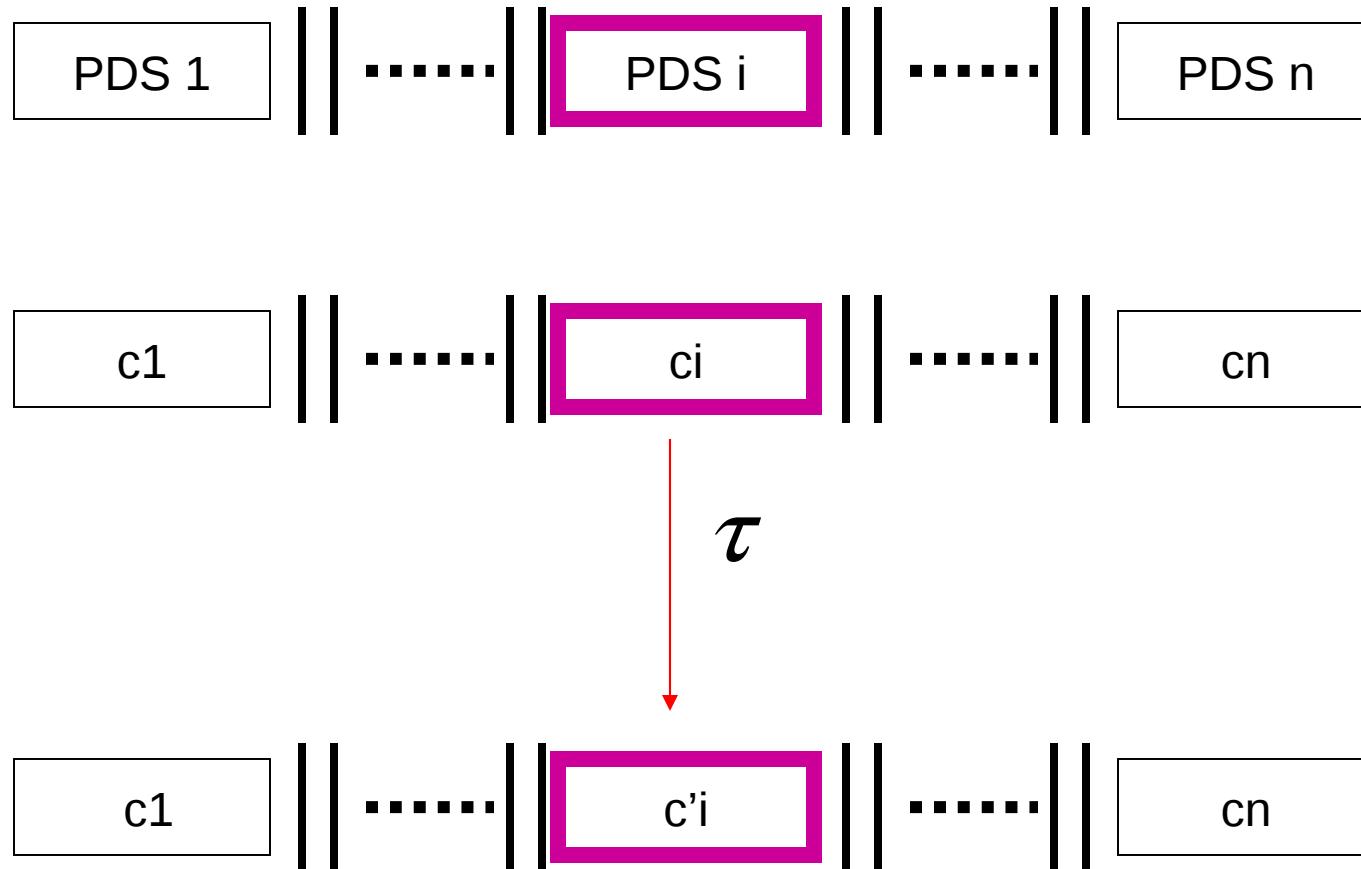
$n$  sequential components running in parallel, communicating via rendez-vous through blocking synchronizing actions



**Communicating Pushdown System (CPDS)**

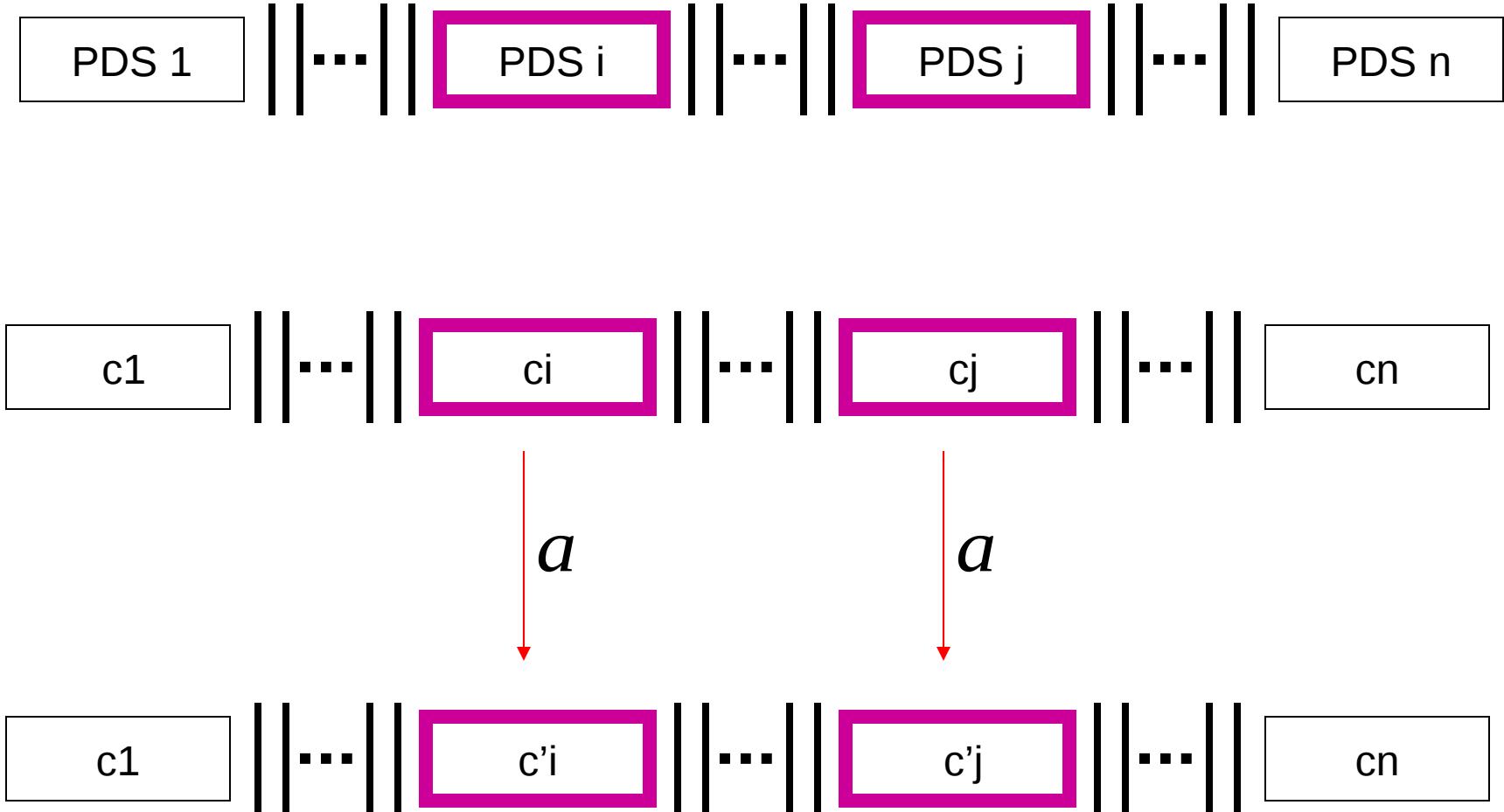
# Communicating Pushdown Systems

## Internal actions $\tau$



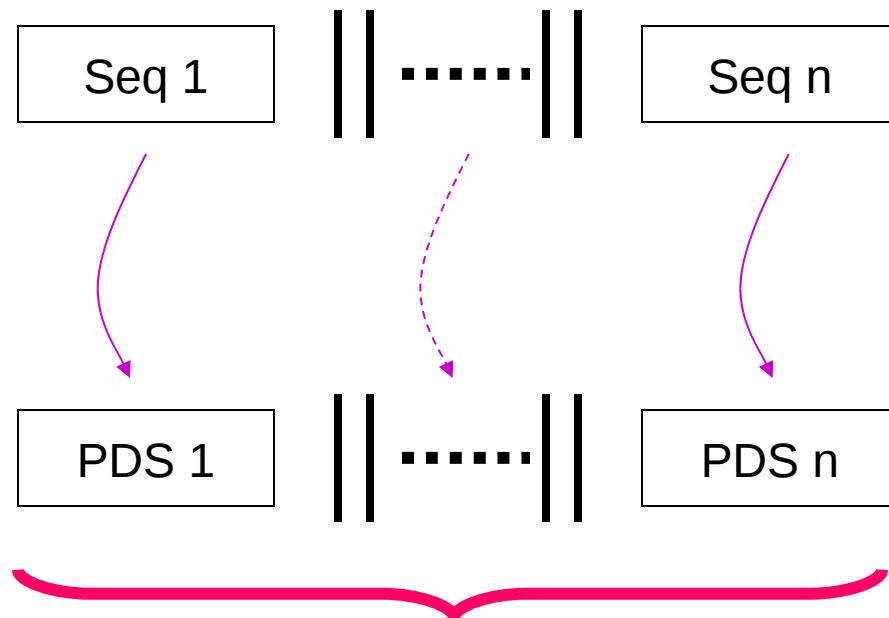
# Communicating Pushdown Systems

## Synchronizing actions



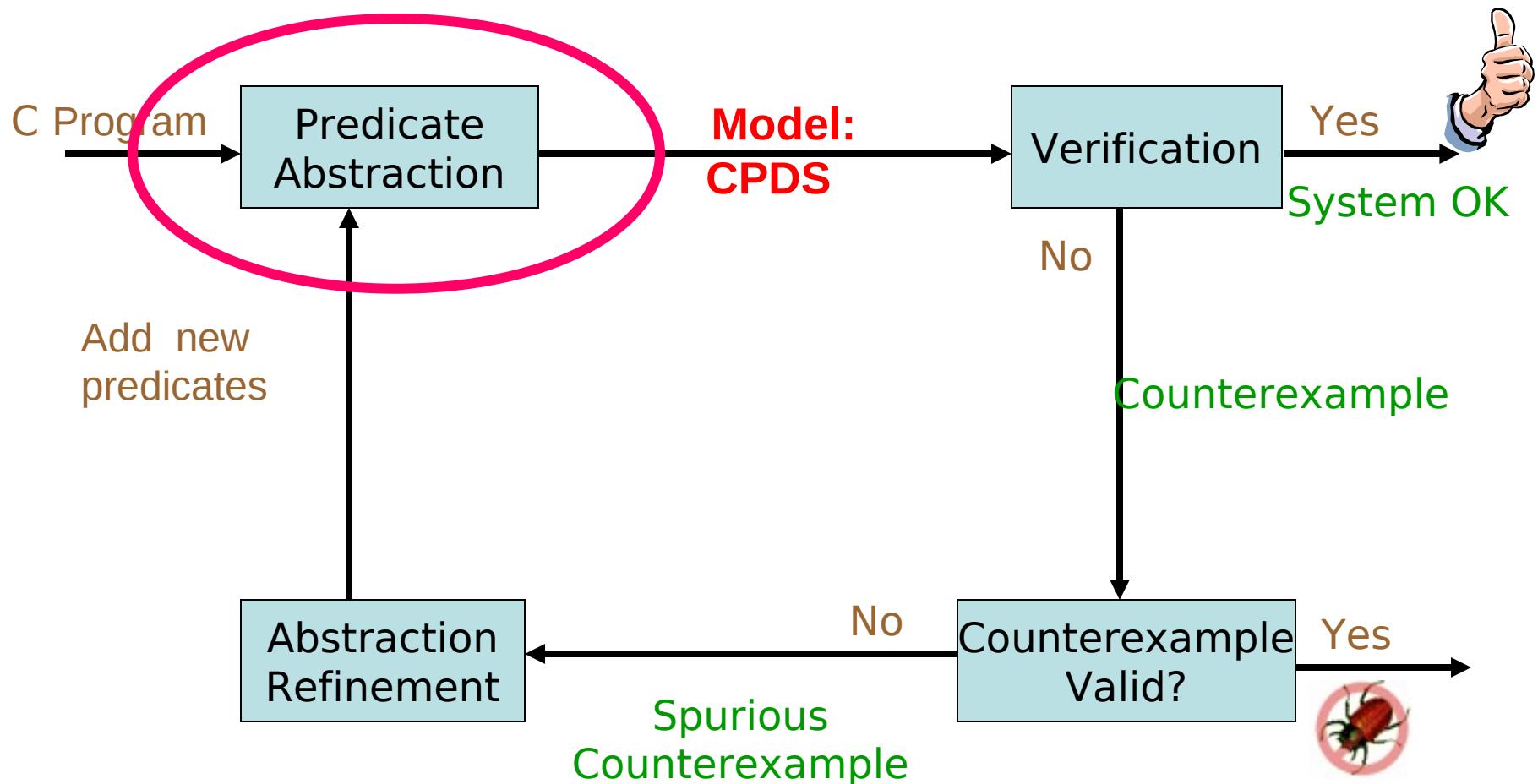
# What about Concurrency?

$n$  sequential components running in parallel, communicating via rendez-vous through synchronizing actions



Communicating Pushdown System (CPDS)

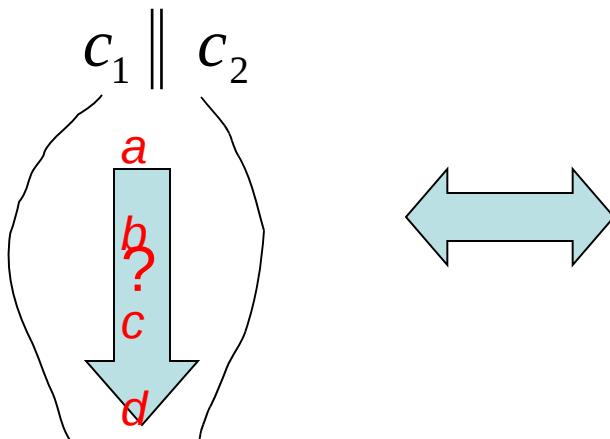
# Our Approach



# Reachability Analysis of CPDSS

PDS 1 || PDS 2

$$L_1 \cap L_2 \neq \emptyset$$



$L_i$  : paths of PDS<sub>i</sub> leading from  $c_i$  to  $c'_i$   
: Context - free language

**Undecidable!**



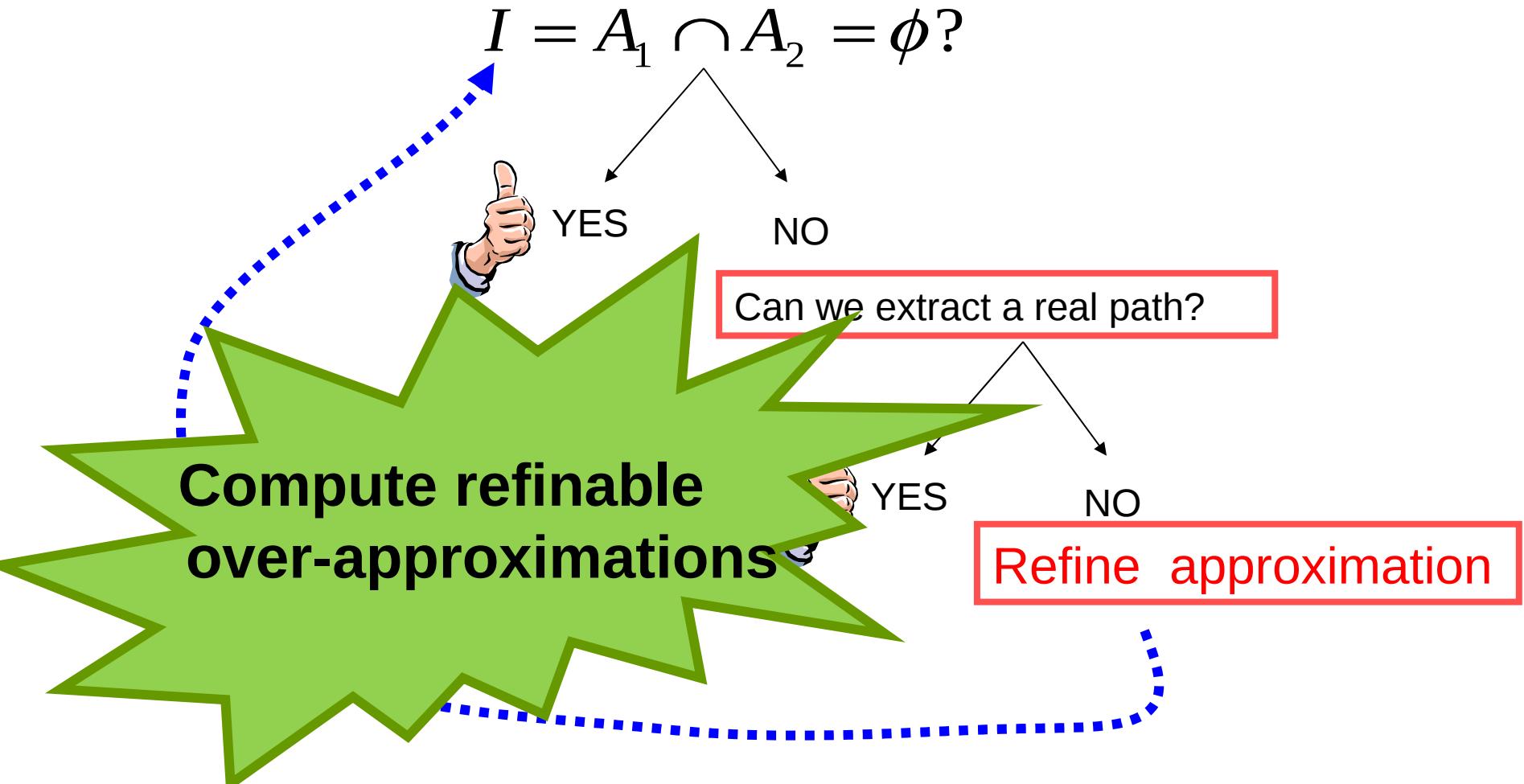
**No exact solution**

# Solution: A CEGAR Scheme

[Clarke,Chaki,Kidd,Reps,Touili'06]

$$L_1 \cap L_2 = \emptyset ??$$

Compute over - approximations  $A_1 \supseteq L_1$  and  $A_2 \supseteq L_2$



# Computing refinable over-approximations

$$\alpha_k(L) = \text{prefix}_k(L) \cdot \Sigma^*$$

Example:  $L = abababc^*$

$$\alpha_3(L) = aba(a + b + c)^*$$

$$\alpha_4(L) = abab(a + b + c)^*$$

Refinable abstractions:  $\alpha_1, \alpha_2, \alpha_3, \dots$

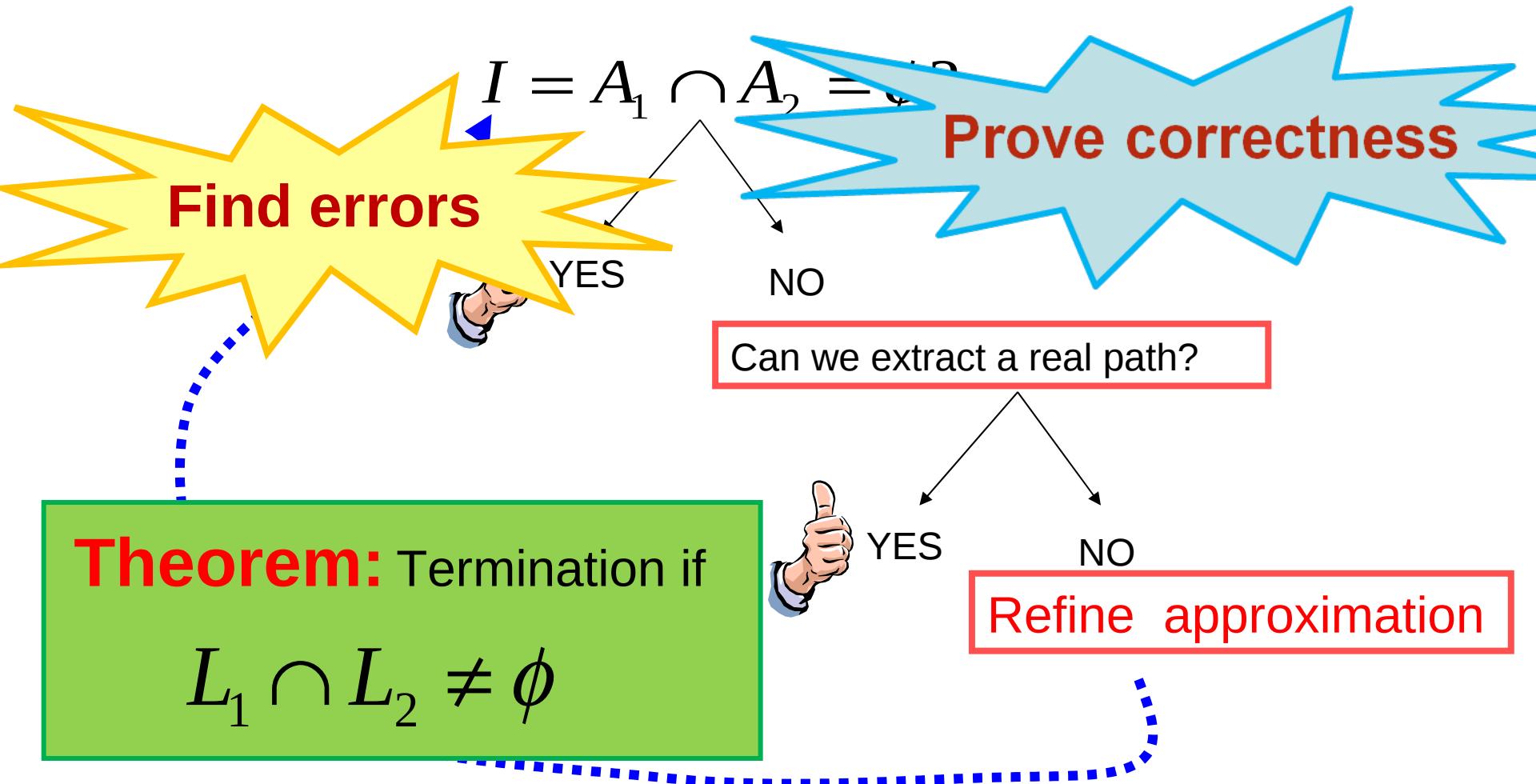
Theorem: [Bouajjani,Esparza,Touili'03]

$L$  : context free language  $\Rightarrow \alpha_k(L)$  can be effectively computed

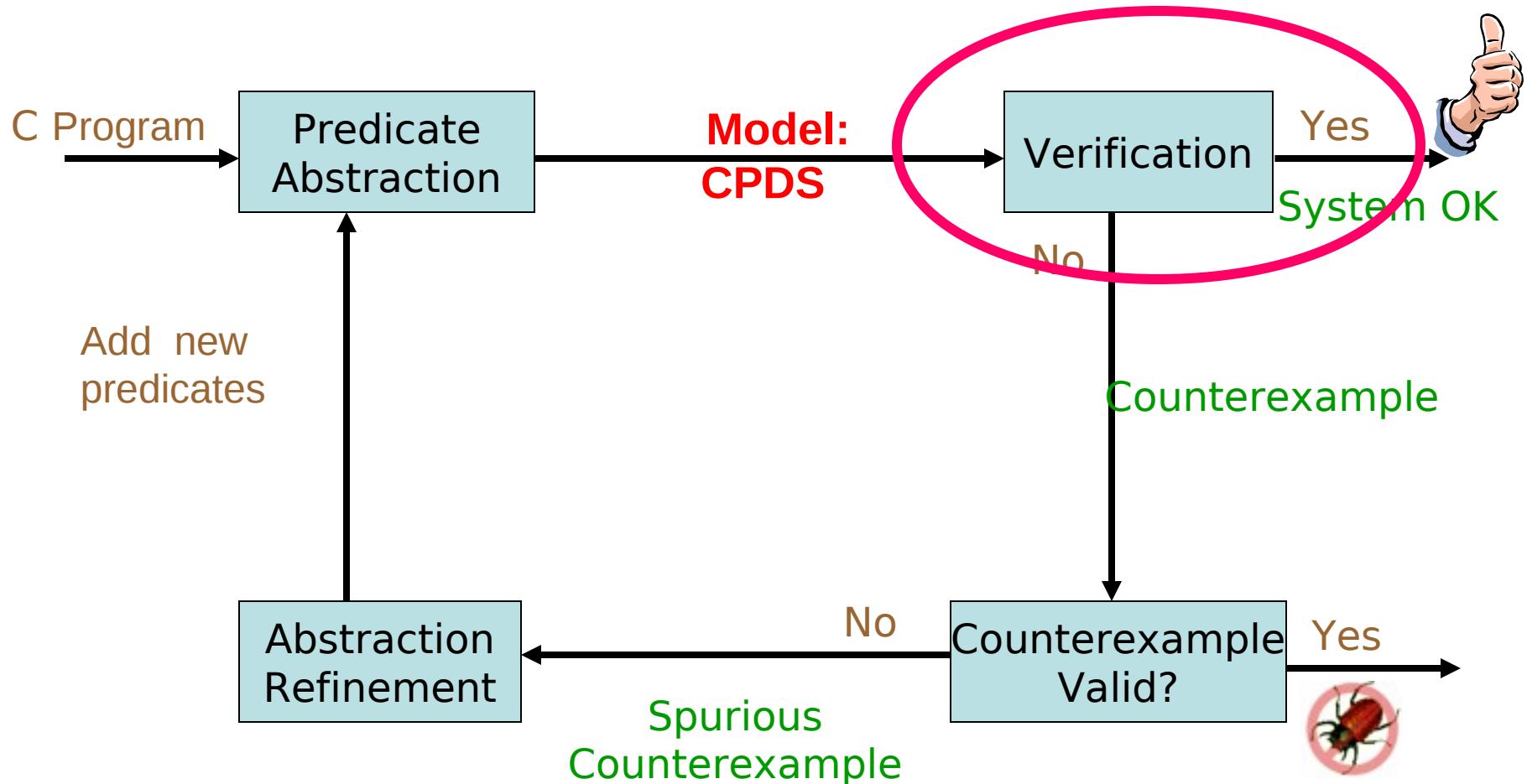
# Solution: A CEGAR Scheme

$$L_1 \cap L_2 = \emptyset ??$$

Compute over - approximations  $A_1 \supseteq L_1$  and  $A_2 \supseteq L_2$

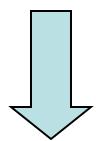


# Our Approach



# CounterExample Validation

Program 1 || Program 2



PDS 1 || PDS 2

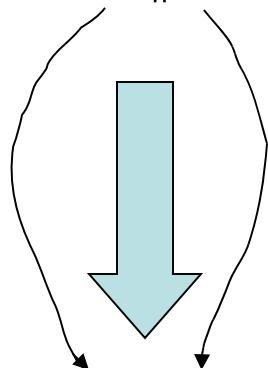
$r_1 \dots r_n$  in PDS1

$r'_1 \dots r'_m$  in PDS2

$c_1 \parallel c_2$

$s_1 \dots s_n$  in Program1?

$s'_1 \dots s'_m$  in Program2?



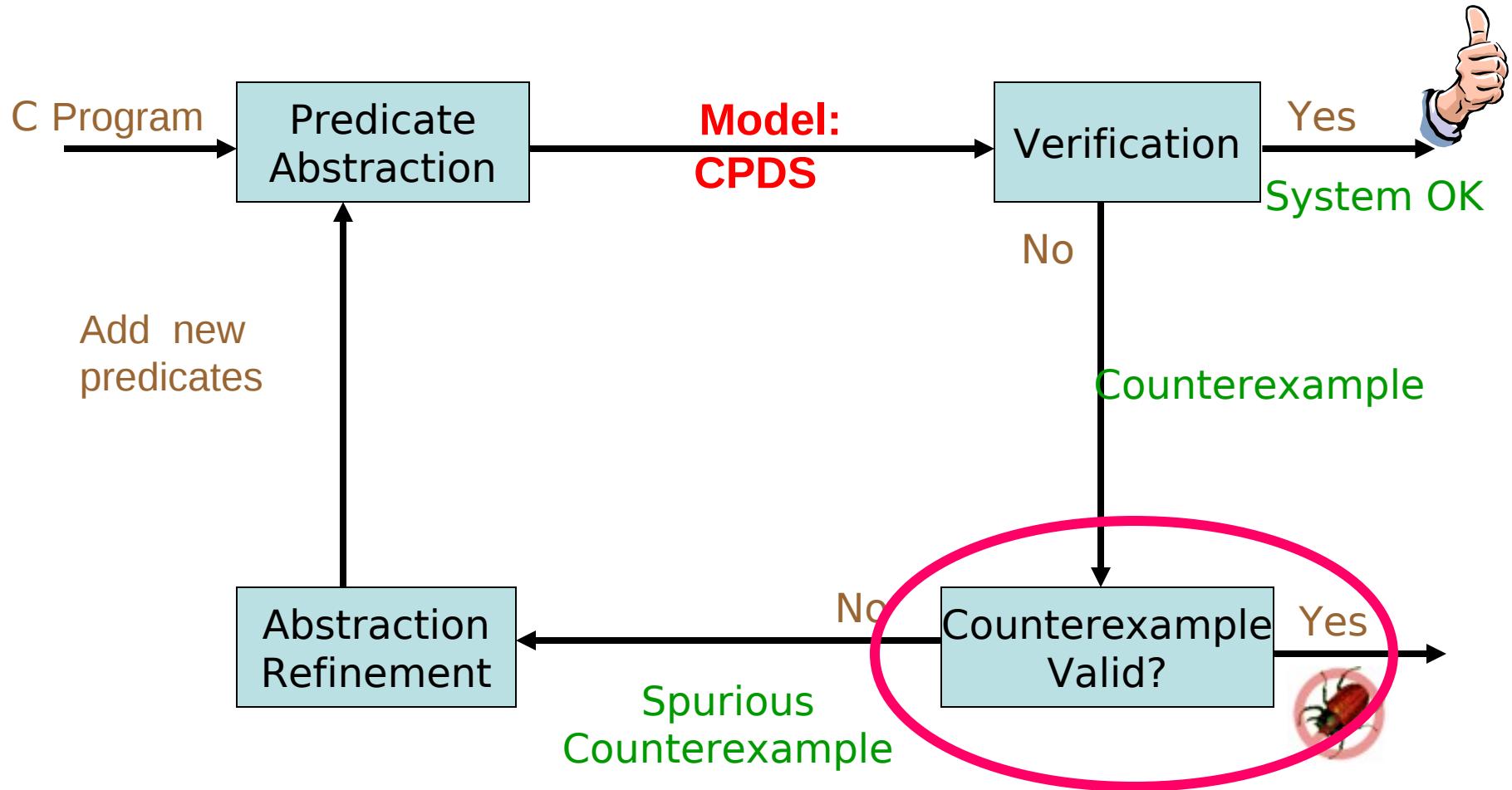
$c'_1 \parallel c'_2$

# CounterExample Validation

$s_1 \dots s_n$  in Program1?

- Initially,  $\varphi = \text{glob}_0 \wedge \text{loc}_0$
- For  $i=1$  to  $n$  do:
- If  $s_i$  assignment, compute the strongest postcondition of  $\varphi$  w.r.t.  $s_i$
- $s_i : x := x + 5; \quad \varphi : 1 < x < 4; \quad \varphi' : 6 < x < 9$
- If  $s_i$ : if statement with condition  $c$ ;  $\varphi' : \varphi \wedge c$
- If  $\varphi$  satisfiable, the counterexample is valid

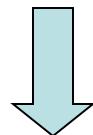
# Our Approach



# Abstraction Refinement

$s_1 \dots s_n$  not in Program1

Process 1 || Process 2



Refine PDS1

PDS 1 || PDS 2

Add new conditions in C

# Implementation

# MAGIC



- Modular Analysis of proGrams In C  
<http://www.cs.cmu.edu/~chaki/magic>
- A Model checker for Concurrent C-programs developed at CMU
- Counter-Example Guided Abstraction Refinement (CEGAR)

# Experiments: Windows NT Bluetooth driver

- Discovery of a new **unknown** bug in a corrected version **(20 seconds)**
- Re-discovery of a bug in an old version **(5 seconds)**

# Experiments: Concurrent Insertions in Binary trees

- Find a bug in an uncorrect version

#processes	time(s)
2	0.8
3	0.8
4	0.8
5	1.1
6	2.7
7	12.9

# Experiments

## Non-recursive Programs

- Our technique behaves better than inlining

OpenSSL Server1	16.2	2.82
OpenSSL Server2	16.1	3.83
OpenSSL Server3	14.0	19.2
OpenSSL Server4	14.2	2.76
OpenSSL Server5	14.0	3.02
OpenSSL Server6	14.0	2.93
OpenSSL Server7	15.0	3.34
2MicroC	578.0	1.324
3MicroC	*	2.144
	INLINING	CPDS

# Conclusion

- Techniques+tool for the verification of concurrent recursive programs with unbounded data
- CEGAR in 2 levels: Abstraction level and the verification level
- Technique compositional: scalable to large programs
- Encouraging experimental results  
(discovery of an unknown bug in Windows NT driver+ better execution times)

# Questions?