# Electronic Voting: Design, attacks and Formal Verification

Véronique Cortier, CNRS, Loria (Nancy, France)

Joint work with Bruno Blanchet, Vincent Cheval, Alexandre Debant, Pierrick Gaudry, Stéphane Glondu, Lucca Hirschi, Léo Louistisserand, Florian Moser

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### Internet voting is used in various countries

- ► France: National Assembly, for expats only (2012, 2022, 2024)
- ► Estonia: local elections (since 2005), national parliamentary elections (since 2007), more than 50% of votes cast by Internet in 2023
- ► Australia: New South Wales state (2021, more than 650 000 votes cast by Internet)
- ► Switzerland: several trials, a demanding and evolving regulation since 2013
- ► Canada: local election in Ontario (since 2003) and Nova Scotia (since 2006)

## Widely used in non-political election

- professional elections
- associations
- administration councils
- scientific councils

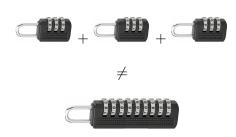


#### Elections in Moscow, in 2019 [P. Gaudry]

- ▶ ballots posted on a blockchain (why?)
- bug bounty program



3 keys of 256 bits  $\neq$  1 key of 768 bits



#### Swiss context

- ▶ open specification, open source code
- call for public scrutiny
- multiple elections in one round



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#### And also

- bad https channel in Australian elections
- complete take-over in overseas US military elections
- ► PacMan installed on Sequoia Machines AVC Edge
- tampering on voting machines in India
- ▶ ..

/39

What is a good voting system?

Vote privacy
"No one should know how I voted"



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Better: Receipt-free / Coercion-resistant
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- vote buying
- coercion





#### Vote privacy

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Everlasting privacy: no one should know my vote, even when the cryptographic keys will be eventually broken.

#### Verifiability

#### Individual Verifiability: a voter can check that

- cast as intended: their ballot contains their intended vote
- recorded as cast: their ballot is in the ballot box.

#### Universal Verifiability: everyone can check that

- ▶ tallied as recorded: the result corresponds to the ballot box.
- eligibility: ballots have been casted by legitimate voters.



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#### Even better: accountability

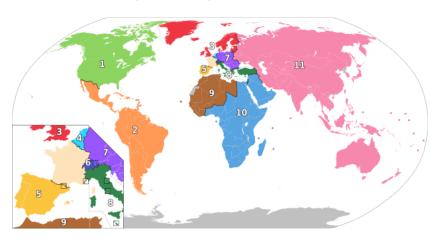
- ▶ the system tells whom to blame
- eases dispute resolution

### And many more properties

- Availability: servers available at any time
- ► Accessibility: easy to use, adapted to people with various issues
- **.**.

### 2022 French legislative elections

11 circonscriptions (11 deputies), 1.6 M voters.



Crédits: Pierre-Yves Beaudouin / Wikimedia Commons / CC BY-SA 3.0

#### Context

#### Voters can vote:

- by postal mail
- ▶ at a polling station (at the consulate)
- by Internet

#### Security level required:

Level 3 (the highest) of the CNIL recommendations

This implies verification by **third party tools**.

Objectif de sécurité n° 3-02 : Permettre la transparence de l'urne pour tous les électeurs à partir d'outils tiers.

Building blocks: cryptography

### Threshold decryption

- ► Each trustee computes her secret key
- ► The *n* trustees jointly compute the public key pk
- Decryption with t out of the n keys: t out of n trustees suffice to produce decryption shares, that yield the plaintext

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ightarrow The decryption key is never present on a single computer, neither during the key generation nor the decryption!

#### Zero-Knowledge proofs



#### Examples

▶ Possibility to prove that an encrypted message is either *a* or *b* 

$$\{m\}_k$$
 Proof  $(m = a \text{ or } m = b)$ 

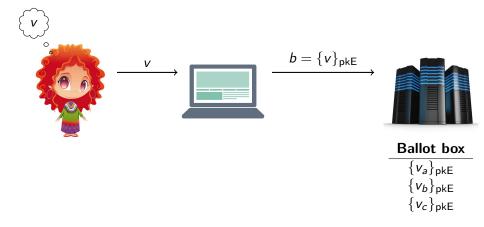
Possibility to prove that the decryption is correct

$$c, m \quad Proof(dec_k(c) = m)$$

14/39

## How the FLEP protocol (should) work

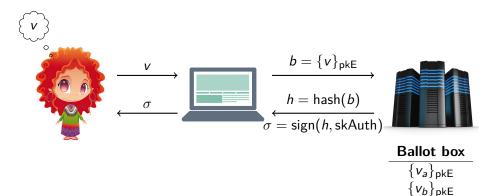
Phase 1: vote for v = 0 or 1



pkE: public key, the private keys are shared among the authorities.

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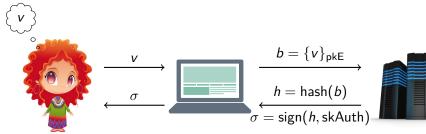


 $\{v_c\}_{pkE}$  $\{v\}_{pkE}$ 

pkE: public key, the private keys are shared among the authorities.

# How the FLEP protocol (should) work

Phase 1: vote for v = 0 or 1





# Phase 2: Tally - homomorphic encryption (El Gamal)

$$\{v_1\}_{\mathsf{pkE}} \times \cdots \times \{v_n\}_{\mathsf{pkE}} = \{v_1 + \cdots + v_n\}_{\mathsf{pkE}}$$

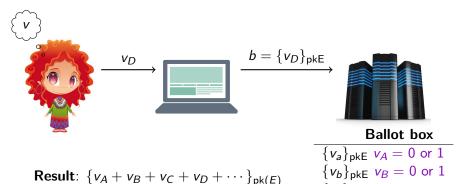
since  $g^a \times g^b = g^{a+b}$ 

→ Only the final result needs to be decrypted! And proved.

Ballot box  $\{v_a\}_{\mathsf{pkE}}$  $\{v_b\}_{pkE}$  $\{v_c\}_{pkE}$  $\{v\}_{\mathsf{nkE}}$ 

pkE: public key, the private keys are shared among the authorities.

# A closer look at ballots - validity

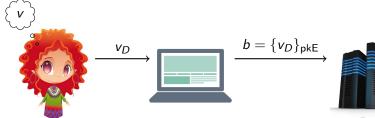


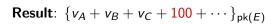
 $\{v_c\}_{pkE} \ v_C = 0 \text{ or } 1$ 

 $\{v_D\}_{pkE}$ 

16/39

# A closer look at ballots - validity



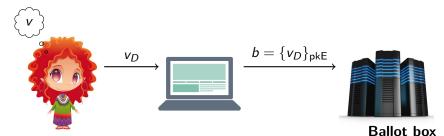


A voter could cheat!

# Ballot box

 $\{v_a\}_{pkE} \ v_A = 0 \text{ or } 1$  $\{v_b\}_{pkE} \ v_B = 0 \text{ or } 1$  $\{v_c\}_{pkE} \ v_C = 0 \text{ or } 1$  $\{v_D\}_{pkE} \ v_D = 100$ 

# A closer look at ballots - validity



 $\{v_a\}_{pkE}$   $v_A = 0$  or 1 $\{v_b\}_{pkE}$   $v_B = 0$  or 1

 $\{v_c\}_{pkE} \ v_C = 0 \text{ or } 1$  $\{v_D\}_{pkE} \ \frac{v_D = 100}{}$ 

**Result**:  $\{v_A + v_B + v_C + v_D + \cdots\}_{pk(E)}$ 

A voter could cheat!

Use a zero-knowledge proof

$$\{v_D\}_{\mathsf{pk}(E)}, \mathsf{Proof}\{v_D = 0 \text{ or } v_D = 1\}$$

16/39

### A closer look at ballots - multiple candidates

4 candidates: A, B, C, D Assume Alice wants to vote for C

candidates	Α	В	C	D
vote	0	0	1	0
ballot	$\{0\}_{pkE}$	$\{0\}_{pkE}$	$\{1\}_{pkE}$	$\{0\}_{pkE}$

$$+ \operatorname{Proof}\{v_A=0 \text{ or } v_A=1\}, \ldots, \operatorname{Proof}\{v_D=0 \text{ or } v_D=1\}$$
  $+ \operatorname{Proof}\{v_A+v_B+v_C+v_D=1\}$ 

17/39

#### Voter receipt

#### Elections législatives 2022 1er tour



#### Preuve de dépôt du bulletin de vote dans l'urne

Voici la preuve de dépôt de votre bulletin dans l'urne.

Votre bulletin de vote a bien été introduit dans l'urne électronique.

La référence ci-dessous vous permet de contrôler que votre bulletin est bien dans l'urne.

80011&1&3318f83ea80861c9e6274f049c8df87c2da4fe03e43b7aa46b71 92c0cfc3129c53

Pour contrôler la référence de votre bulletin : cliquez ici https://votefae.diplomatie.gouv.fr/pages/verifierEmpreinte

Une fois le dépouillement effectué, vous pouvez vérifier que votre bulletin a bien été pris en compte dans le calcul des résultats, à l'aide d'un outil tiers développé par le CNRS, conformément aux exigences de la CNIL en matière de transparence de l'urne. Pour ce faire, vous devrez renseigner le cachet électronique ci-dessous.

Vous pouvez accédez à l'outil en cliquant ici.

Ce cachet électronique vous permet également de vérifier que votre preuve de vote a bien été produite par le système de vote homologué.



eyJphar2UIU3014MhAxdWxxfDflcmVfQ2lyY29uc2NyxKBOaN9uXZR1c196cmFv2Fpc19kZV9s372V0cmFv2ZVyfDE
WXXwxH2E41gyzZMEAH0gZMMND5TYNP.ExmNDG7xhk2gj37x1AFTmETAT2fC7gyfjdfYD2YjcySyJMKDMrv2MyJMCDMrv2WyfDE
UZIIwic2Nobm9yc1161jFraWsxaTV2OHQzMG3NXZhb\_ZhtMMhic292aTc1bGE5YMQ2cXBsbmNodX2lajU5c2cub18ljoi
HX8dJVXM95zg5yMcGqMNtoWuXaZMXMDFSMKRWMKOUT072a2wam5qMcfmhwTymWgdcIalmlYmxpY01eVH18ljoi
L80tL81CRUdTI9MRVJJRklDQVXJTNS55SVZLSCLSLcLsLxuNxdmYTMOZTQVYWXZG14ZDbxXMDjMmQ4Y2U0ODNNhic
OTMYYTMFOTTOMHAME1VZRNNjmAnhamMf97gMITOMJOGUCHJYTMWWHITUXMxYZOMXJMZEZYZMYJTNIZMMZYQ1NbMUJWALNIJYSKSOTFNNDMMZQWZTQTOWYJNJM3OTVNXHJCblOtL80tRU5EXIZFUKlGSUNBVEIPT19LRVkLL80tL8IsImNsZ
UMYX2ll6ZJWXQ1011yOS39

### What we did: universal verifiability

Joint work with P. Gaudry and S. Glondu [EVoteID'23]

- Requirement to work on public specifications
- ► No NDA, responsible disclosure instead

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#### After the tally

- ▶ we receive the ballot box
- we check the zero-knowledge proofs of correct decryption (and validity of the ballots)
- with our own software, written independently



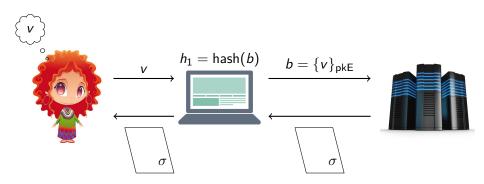
### What we did: individual verifiability

- ▶ During the voting phase: Verification tool for the validity of the server signature;
- ▶ **After the tally:** Publication of the list of hashed ballots + verification tool for checking the presence of a hash in this list.



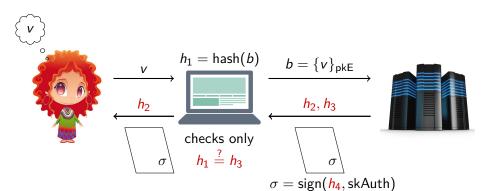
### What we missed: several flaws!

A. Debant, L. Hirschi [Usenix'24]



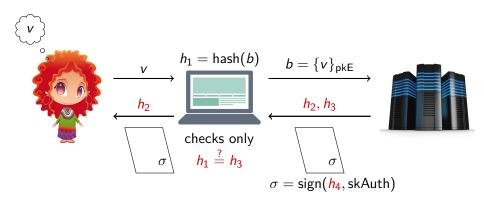
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An unsatisfying fix: now both  $h_1$  and  $h_3$  are displayed to the voter for comparison, while the voting client already checks  $h_1 = h_3$ .

ightarrow The voter needs to check themself that  $h_1=h_4$ , without any instruction.

### Formal analysis of e-voting systems

Why a formal analysis of an e-voting system?

 $\longrightarrow$  Because formal methods can find attacks before implementations

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22/3

### Formal analysis of e-voting systems

Why a formal analysis of an e-voting system?

- $\longrightarrow$  Because formal methods can find attacks before implementations
- → Now a current practice for many protocols (TLS, 5G, ...)
- $\rightarrow$  Legal requirements in Switzerland to provide symbolic and  $cryptographic\ proofs$  of e-voting protocols.

#### 5.1. Examining the cryptographic protocol

5.1.1 Examination criteria: The protocol must meet the security objective according to the trust assumptions in the abstract model in accordance with Section 4. In addition, a cryptographic and a symbolic proof must be provided. The proofs relating to cryptographic basic components may be provided according to generally accepted security assumptions (for example, the 'random oracle model', 'decisional Diffiel-Hellman assumption', 'Fiat-Shamir heuristic'). The protocol should be based if possible on existing and proven protocols.

## Two main models for security

	Formal approach	Computational approach
Messages	$\{\}$ $\langle , \rangle$ $k$ $A$ $N_A$	0101000101110101 1101010110101010 001110101110110
Encryption	terms	algorithm
Adversary	idealized	any polynomial algorithm
Guarantees	some attacks missed	stronger
Proof	often automatic	mostly by hand difficult for complex protocols

### Messages

Messages are abstracted by terms.

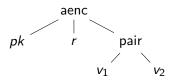
Agents :  $a, b, \ldots$  Nonces :  $n_1, n_2, \ldots$ 

Keys:  $k_1, k_2, \ldots$ 

Ciphertext : aenc(pk, r, m) Concatenation :  $pair(m_1, m_2)$ 

denoted simply  $(m_1, m_2)$  in ProVerif

Example: The encrypted message  $aenc(pk, r, pair(v_1, v_2))$  is represented by:



Intuition: only the structure of the message is kept.

### Model for cryptographic primitives

#### Projection

$$\pi_1(\mathsf{pair}(x,y)) = x$$
  
 $\pi_2(\mathsf{pair}(x,y)) = y$ 

#### Asymmetric and symmetric encryption

$$adec(aenc(pk(y), z, x), y) = x$$
  
$$dec(enc(x, y), y) = x$$

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Asymmetric and symmetric encryption

$$adec(aenc(pk(y), z, x), y) = x$$

$$dec(enc(x, y), y) = x$$

Zero knowledge proof: proof of valid vote

$$aenc(pk, r, m), ZKP(m = 0 OR m = 1)$$

 $Valid(\mathsf{ZKP}(\mathsf{aenc}(\mathsf{pk},r,0),\mathsf{pk},r),\mathsf{aenc}(\mathsf{pk},r,0),\mathsf{pk}) = \mathsf{ok} \\ Valid(\mathsf{ZKP}(\mathsf{aenc}(\mathsf{pk},r,1),\mathsf{pk},r),\mathsf{aenc}(\mathsf{pk},r,1),\mathsf{pk}) = \mathsf{ok} \\$ 

### Syntax for processes

The grammar of processes is as follows:

```
P,Q,R:= 0

if M_1=M_2 then P else Q

let x=M in P

in(c,x);P

out(c,N);P

new n;P

P\mid Q

!P

eventE.P
```

Syntax of ProVerif, a dialect of the applied-pi calculus [AbadiFournet01]

### ProVerif: automatic analysis of protocols

#### Developed by Bruno Blanchet and Vincent Cheval

### Performs very well in practice!

- Works on most of existing protocols in the literature
- Is also used on industrial protocols (e.g. TLS, Signal, ...)
- used to pass Swiss requirements on voting
  - ► Neuchâtel/Scytl protocol [C., Turuani 2018]
  - ► CHVote protocol [C., Turuani 2019]
  - ► Swiss Post [Debant, C., Gaudry, 2022→now]

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- $\rightarrow$  ProVerif translates processes in applied pi-calculus into Horn clauses (first-order logic).

#### Attacker

Horn clauses perfectly reflects the attacker symbolic manipulations on terms.

$$\begin{array}{llll} \forall x \forall y & l(x), l(y) & \Rightarrow & l(\operatorname{enc}(x,y)) & \operatorname{encryption} \\ \forall x \forall y & l(\operatorname{enc}(x,y)), l(y) & \Rightarrow & l(x) & \operatorname{decryption} \\ \forall x \forall y & l(x), l(y) & \Rightarrow & l(x) & \operatorname{concatenation} \\ \forall x \forall y & l(x), l(y) & \Rightarrow & l(x) & \operatorname{first projection} \\ \forall x \forall y & l(x), l(y) & \Rightarrow & l(y) & \operatorname{second projection} \\ \forall x \forall y & l(x), l(y) & \Rightarrow & l(y) & \operatorname{second projection} \\ \end{array}$$



### Protocol as Horn clauses

```
let Voter(pkE, Vote, id, cauth) =

new r: bitstring;

let b = (id, aenc(pkE, r, Vote))

event Voted(id, Vote, r)

out (cauth, b);

out (c, b).
```

Each action of the protocol is translated into logical implications.

$$\forall v \quad l(v) \Rightarrow l(\langle id, \mathsf{aenc}(\mathsf{pkE}, r(v), v \rangle)$$

$$\forall v \quad l(v) \Rightarrow \mathsf{Voted}(id, v, r(v))$$

# Security reduces to consistency



secure?

⌇

```
 \forall x \forall y \qquad I(x), I(y) \Rightarrow I(< x, y >) 
\forall x \forall y \qquad I(x), I(y) \Rightarrow I(\text{enc}(x, y)) 
\forall x \forall y \qquad I(\text{enc}(x, y)), I(y) \Rightarrow I(x) 
\forall x \forall y \qquad I(< x, y >) \Rightarrow I(x) 
\forall x \forall y \qquad I(< x, y >) \Rightarrow I(y) 
\forall v \qquad I(v) \Rightarrow I(\langle id, \text{aenc}(\text{pkE}, r(v), v \rangle) 
\forall v \qquad I(v) \Rightarrow \text{Voted}(id, v, r(v))
```

## Security reduces to consistency



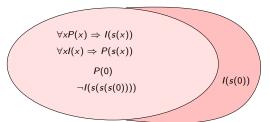
secure?

{

Does not yield a contradiction?

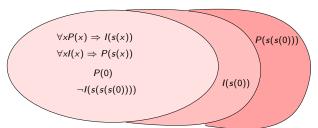
(i.e. consistent theory ?)

Idea: add logical consequences . . .



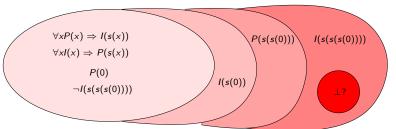
... until a contradiction is found.

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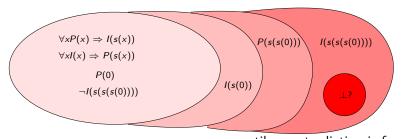
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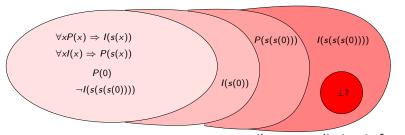


... until a contradiction is found.

Ideally, we need a method (a strategy) which is:

- correct: adds formula that are indeed consequences
- complete: finds a contradiction (if it exists)
- ▶ in a finite number of steps

Idea: add logical consequences ...

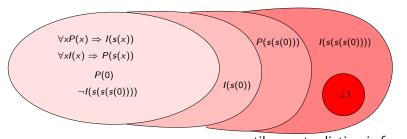


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- in a finite number of steps undecidable fragment

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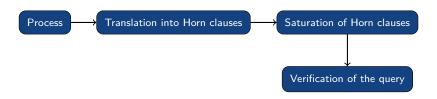
... until a contradiction is found.

Ideally, we need a method (a strategy) which is:

- correct: adds formula that are indeed consequences
- complete over-approximations
- in a finite number of steps undecidable fragment

#### **ProVerif**

- ► Implements a correct procedure (that may not terminate or just stop without answer).
- ▶ Based on a resolution strategy well adapted to protocols.



### Binary resolution

$$\frac{H \Rightarrow C \quad F, H' \Rightarrow C'}{H\sigma, H'\sigma \Rightarrow C'\sigma}$$
 with  $\sigma$  substitution s.t.  $C\sigma = F\sigma$ 

- correct
- but adds too many clauses (never terminates)

### Binary resolution

$$\frac{H \Rightarrow C \quad F, H' \Rightarrow C'}{H\sigma, H'\sigma \Rightarrow C'\sigma} \text{ with } \sigma \text{ substitution s.t. } C\sigma = F\sigma$$
$$F \neq I(x)$$

- correct
- but adds too many clauses (never terminates)

#### ProVerif's strategy:

- ightharpoonup do not resolve on I(x)
- ▶ well crafted resolution strategy

1. Horn clauses yield over-approximations Example: non uniqueness  $\forall v \mid l(v) \Rightarrow Voted(id, v, r(v))$ 

1. Horn clauses yield over-aproximations

```
Example: non uniqueness \forall v \mid (v) \Rightarrow \mathsf{Voted}(id, v, r(v)) yields \mathsf{Voted}(id, v_1, r(v_1)), \mathsf{Voted}(id, v_2, r(v_2)), \dots
```

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Idea: axioms

 $Voted(id, v_1, r_1), Voted(id, v_2, r_2) \Rightarrow v_1 = v_2 \text{ AND } r_1 = r_2$ 

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Idea: axioms

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Saturation by resolution may still not terminate (despite ProVerif's strategy)

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Example: non uniqueness \forall v \mid (v) \Rightarrow \mathsf{Voted}(id, v, r(v)) yields \mathsf{Voted}(id, v_1, r(v_1)), \mathsf{Voted}(id, v_2, r(v_2)), \dots
```

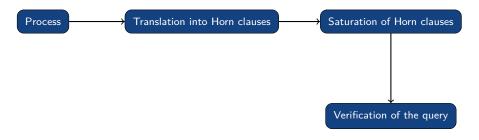
Idea: axioms

$$Voted(id, v_1, r_1), Voted(id, v_2, r_2) \Rightarrow v_1 = v_2 AND r_1 = r_2$$

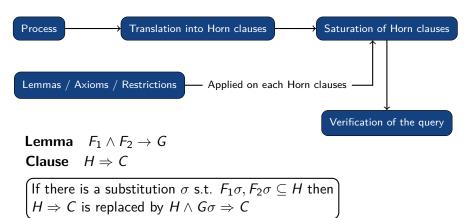
Saturation by resolution may still not terminate (despite ProVerif's strategy)

Idea: lemma as proof helpers

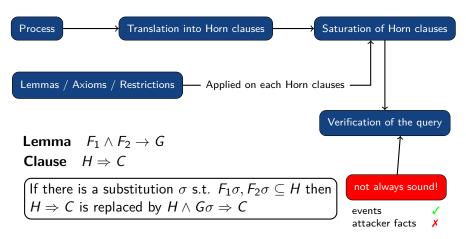
[S&P'22, with B. Blanchet and V. Cheval]



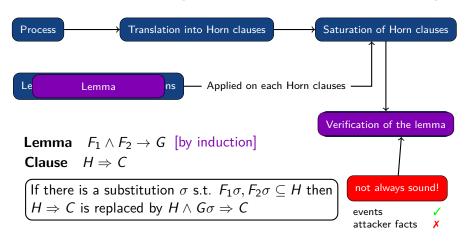
[S&P'22, with B. Blanchet and V. Cheval]



[S&P'22, with B. Blanchet and V. Cheval]



[S&P'22, with B. Blanchet and V. Cheval]



Even better: lemma by induction

### Some challenges

#### Better formal verification

- decision procedures for larger equational theory classes
- better tools
- formalise security properties, possibly identifying new ones

#### Better e-voting systems

- more security properties: no vote buying, everlasting privacy, ...
- less trust assumptions (corrupted computers, ...)
- better authentication

#### Better regulations

- ▶ full public specification → should appear in CNIL 2025!
- third party verification
- clear threat models

