Interrupt Timed Automata

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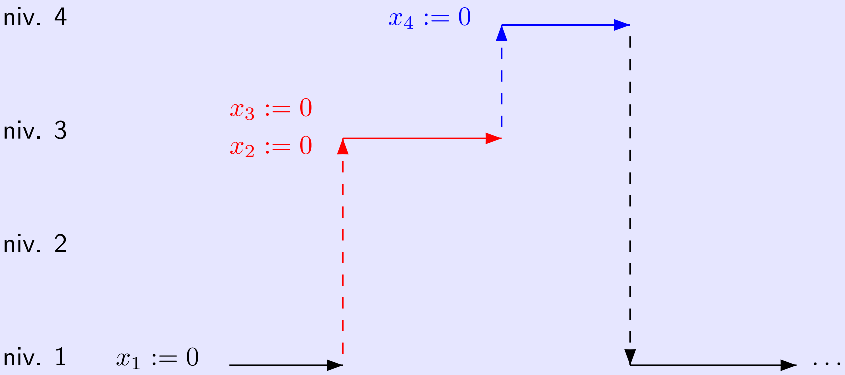
Motivations

- Theoretical: investigate subclasses of hybrid automata with stopwatches, to obtain decidability results in view of negative results, among them:
  - Henzinger et al. 1998: The reachability problem is decidable for rectangular initalized automata, but becomes undecidable for slight extensions, e.g. adding one stopwatch to timed automata.
  - Cassez, Larsen 2000: Linear hybrid automata and automata with stopwatches (and unobservable delays) are equally expressive.
  - Bouyer, Brihaye, Bruyère, Markey, Raskin 2006: Model checking timed automata with stopwatch observers is undecidable for WCTL (a weighted extension of CTL).
- Practical: Many real-time systems include interruptions (as in processors). An interrupt clock can be seen as a restricted type of stopwatch.

Interruptions and real-time

Several levels with exactly one active clock at each level



Execution : $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{1.5} \begin{bmatrix} 1.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{2.1} \begin{bmatrix} 1.5 \\ 0 \\ 2.1 \\ 0 \end{bmatrix} \xrightarrow{1.7} \begin{bmatrix} 1.5 \\ 0 \\ 2.1 \\ 1.7 \end{bmatrix} \xrightarrow{2.2} \begin{bmatrix} 3.7 \\ 0 \\ 2.1 \\ 1.7 \end{bmatrix}$

Outline

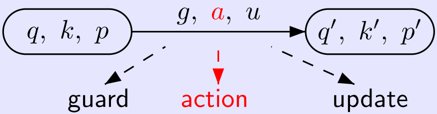
1. ITA model
2. Effective regularity
3. Complexity of the reachability problem
4. Expressiveness
5. Conclusion and perspectives

Outline

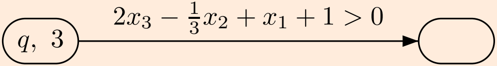
Interrupt Timed Automata (ITA)

$\mathcal{A} = (\Sigma, X, Q, q_0, F, \lambda, pol, \Delta)$

- The mapping λ associates a level in $\{1, \ldots, n\}$ with each state, $x_{\lambda(q)}$ is the active clock in state q
- The mapping pol associates a timing policy with each state: U for urgent, D for delayed and L for lazy
- Transitions in Δ:



Guard: conjunction of linear constraints on clocks from levels $j \leq k$
$\sum_{j=1}^{k} a_j x_j + b \bowtie 0$, with constants in $\mathbb{Q}$



Updates in ITA

From level k to level k'

Increasing level

Clocks of level greater than k' are unchanged, clocks with level from $k+1$ up to k' are reset, and clocks from level less than or equal to k may be updated by a linear expression $x_i : \sum_{j<i} a_j x_j + b$.

Example

Strictly decreasing level

Clocks of level greater than k' are unchanged and all other clocks (including the one at level k') may be updated by a linear expression $x_i := \sum_{j<i} a_j x_j + b$.

Remark: in a state at level k, all clocks from higher levels are irrelevant.

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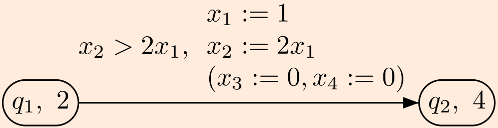

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Example

$$
\begin{tikzcd}[column sep=huge]
(q_1,\ 2) \arrow[r, "{x_2 > 2x_1,\ \begin{array}{l} x_1 := 1 \\ x_2 := 2x_1 \\ (x_3 := 0, x_4 := 0) \end{array}}"] & (q_2,\ 4) \arrow[r, "{x_3 = 3x_1 + x_2,\ \begin{array}{l} x_1 := 0 \\ x_2 := x_1 + 1, \\ x_3 := 2x_2 \end{array}}"] & (q_3,\ 3)
\end{tikzcd}
$$

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Remark: in a state at level k, all clocks from higher levels are irrelevant.

Semantics

For an ITA $\mathcal{A}$

A transition system $\mathcal{T}_{\mathcal{A}} = (S, s_0, \rightarrow)$, with

- configurations $S = \{(q, v, b) \mid q \in Q,\ v \in \mathbb{R}^X, b \in \{\perp, \top\}\}$,
- initial configuration $(q_0, \mathbf{0}, \perp)$,
- transition relation $\rightarrow$

Time step: only the active clock evolves in a state (q, k, p)

- $(q, v, b) \xrightarrow{d} (q, v', \top)$, where $v'(x_k) = v(x_k) + d$ and $v'(x) = v(x)$ for the other clocks.
- If $p = U$, no time step is allowed.

Discrete step:

- $(q, v, b) \xrightarrow{a} (q', v', \perp)$ if there is a transition $q \xrightarrow{\varphi, a, u} q'$ in Δ such that $v \models \varphi$ and $v' = v[u]$.
- If $p = D \wedge b = \perp$, then discrete steps are disallowed.

Language

$\mathcal{L}(\mathcal{A})$ is the set of (finite) timed words associated with a path in $\mathcal{T}_{\mathcal{A}}$ from $(q_0, \mathbf{0})$ to some configuration (q_f, v), for some $q_f \in F$.

ITL : family of languages accepted by ITA.

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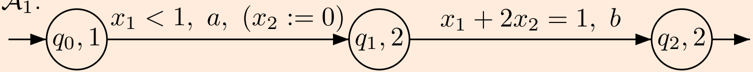
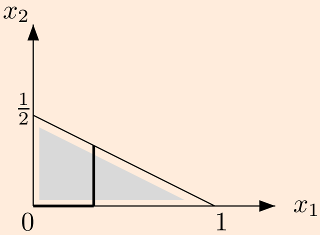
ITL : family of languages accepted by ITA.

Examples

$\mathcal{A}_1$:



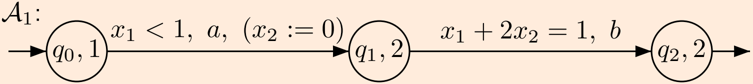
accepts $L_1 = \{(a, 1-\tau)(b, 1-\tau/2) \mid 0 < \tau \leq 1\}$, with trajectories in:



Light gray zone for state q_1:

$(0 < x_1 < 1,\ 0 < x_2 < -\frac{1}{2}x_1 + \frac{1}{2})$

Examples



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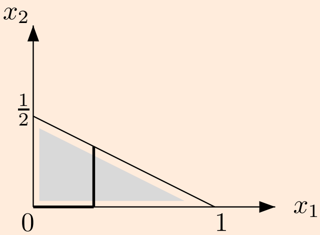

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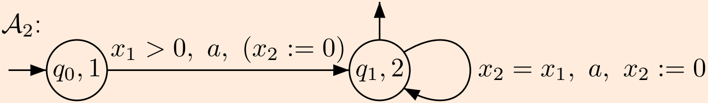

accepts $L_2 = \{(a, \tau)(a, 2\tau) \ldots (a, n\tau) \mid n \in \mathbb{N}, \tau > 0\}$.

Outline

ITA model

2 Effective regularity

Complexity of the reachability problem

Expressiveness

Conclusion and perspectives

A generalized region automaton

Theorem

For a language L in ITL, $Untime(L)$ is effectively regular.

Principle: For an ITA $\mathcal{A} = (\Sigma, X, Q, q_0, F, \lambda, pol, \Delta)$

A finite set $Exp(q)$ of linear expressions is associated with each state $q \in Q$. $Exp(q) = \bigcup_{k \leq \lambda(q)} E_k$, where the sets $E_k = \{0, x_k\}$ are obtained iteratively downward:

- adding the *complements* of x_k in guards from level k,
- saturating E_k by applying updates of appropriate transitions to expressions of E_k,
- saturating E_j $(j < k)$ by applying updates of appropriate transitions to differences of expressions of E_k.

Two valuations are equivalent in state q with level k if they produce the same preorders for linear expressions in each E_i, $i \leq k$.

- A class is a pair $R = (q, \{\preceq_k\}_{k \leq \lambda(q)})$ where $\preceq_k$ is a total preorder on E_k.
- Time successors $R \to R'$ and discrete steps $R \xrightarrow{a} R'$ are then defined.

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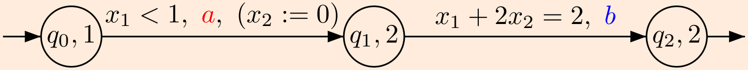
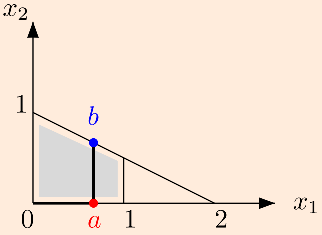
Example

For automaton $\mathcal{A}_3$





$E_1 = \{x_1, 0, 1, 2\}$ and $E_2 = \{x_2, 0, -\frac{1}{2}x_1 + 1\}$

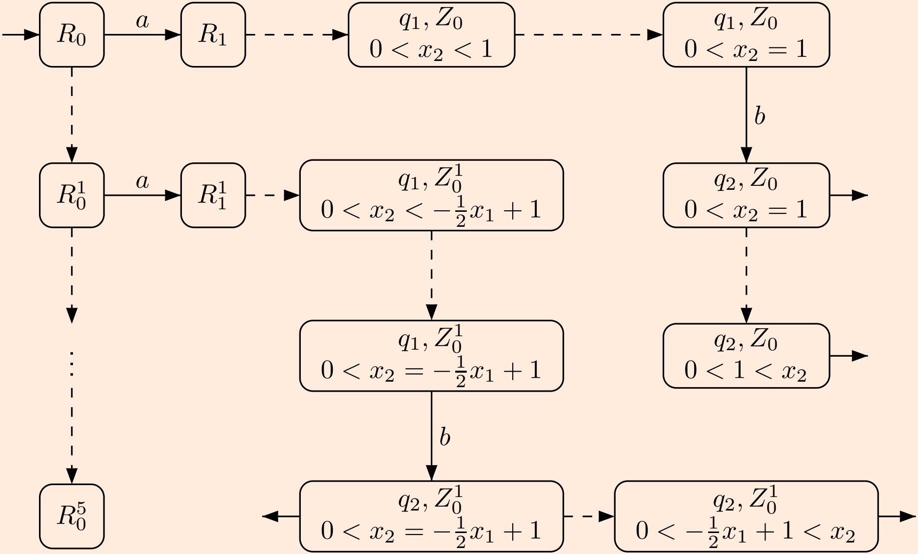
$R_0 = (q_0, Z_0)$ with $Z_0 : x_1 = 0 < 1 < 2$

Time successors of R_0 are $R_0^i = (q_0, Z_0^i)$ with:
$Z_0^1 = (0 < x_1 < 1 < 2)$, $Z_0^2 = (0 < x_1 = 1 < 2)$, $Z_0^3 = (0 < 1 < x_1 < 2)$, $Z_0^4 = (0 < 1 < x_1 = 2)$ and $Z_0^5 = (0 < 1 < 2 < x_1)$

Discrete transitions with action a : $R_0 \xrightarrow{a} R_1 = (q_1, Z_0, x_2 = 0 < \frac{1}{2})$, since $x_1 = 0$, and $R_0^1 \xrightarrow{a} R_1^1 = (q_1, Z_0^1, x_2 = 0 < -\frac{1}{2}x_1 + 1)$

Discrete transitions with action b : from classes such that $x_2 = -\frac{1}{2}x_1 + 1$.

Example (cont.)



Outline

ITA_{-} and reachability

An elementary path in the previous graph can be non deterministically guessed in 2-EXPSPACE leading to the decidability of reachability.

The subclass ITA_{-}

An ITA_{-} is an ITA where updates are restricted to transitions increasing the level, only for the current clock (apart from initializations).

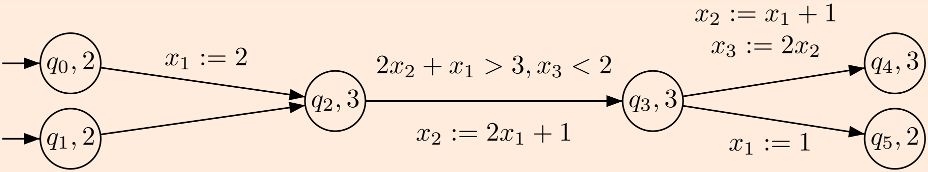
- Reachability in ITA_{-} is decidable in NEXPTIME (existence of an exponentially bounded path).
- An ITA can be transformed into an doubly exponentially larger ITA_{-} with the same clocks accepting the same language.
- Reachability in ITA is decidable in 2-NEXPTIME by combination of these results.
- When the number of clocks is fixed, the reachability problem is NP.

From ITA to ITA_

Principle: Record the forbidden resets in the states

Apply them when needed and use urgent state copies to decrease level.

Example



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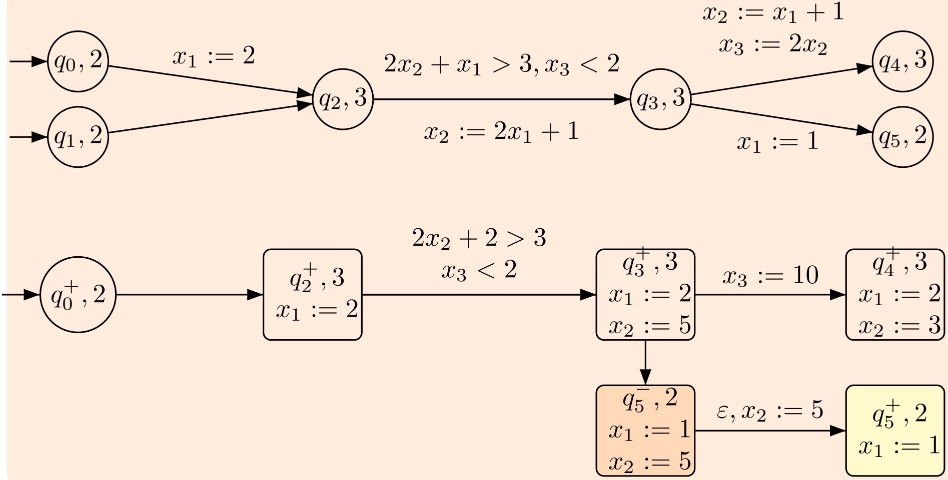

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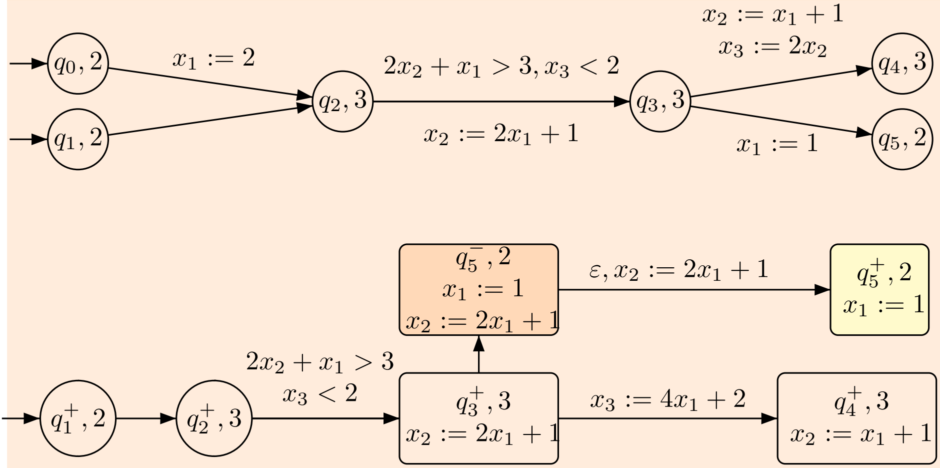
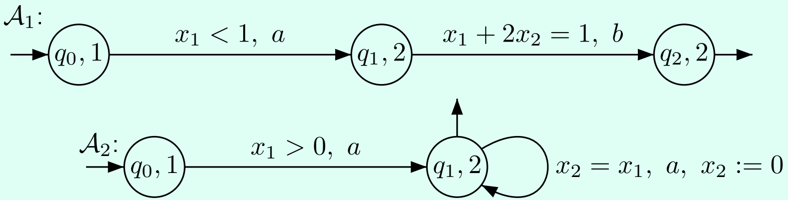


Outline

ITL is neither contained in TL nor in CRTL

Comparing with TA (Timed Automata)

There is no timed automaton accepting L_1 or L_2.



Comparing with CRTA (Controlled Real-Time Automata)

Demichelis, Zielonka, 1998:

There is no controlled real-time automaton accepting L_2.

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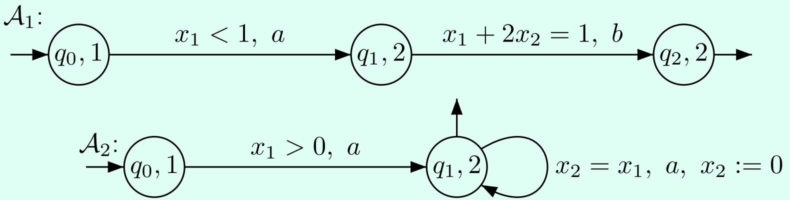

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TL is not contained in ITL

A pumping Lemma

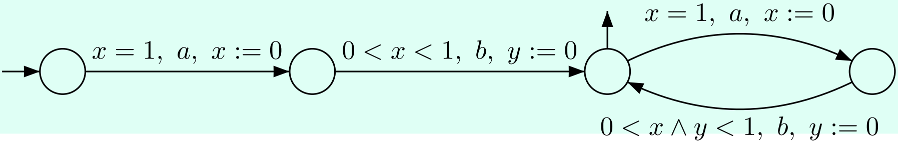
For a language $\mathcal{L}$ in ITL, there exists $B \in \mathbb{N}$ s.t.
given any word (with strictly increasing dates) belonging to $\mathcal{L}$ with B consecutive positions , there are two (**possibly equal**) positions s.t. the subword between these positions can be

- Either duplicated with a **non null** time shift greater or equal than its duration,
- Or erased **without** time shift (in this case the subword is non empty)

and the new word still belongs to $\mathcal{L}$.

ITL and TL are incomparable

A language L_3 accepted by a TA but not by any ITA:



Closure results

ITL is not closed under complement: L_3^c is in ITL

ITL is not closed under intersection : L_3 is the intersection of the two following ITL

- $(a, 1)(b, \tau_1) \ldots (a, n)(b, \tau_n)$ with $\forall 1 \leq i \leq n \; i < \tau_i < i + 1$
- $(a, \tau'_1)(b, \tau_1) \ldots (a, \tau'_n)(b, \tau_n)$ with $\forall 1 \leq i \leq n - 1 \; \tau_{i+1} - \tau_i < 1$

Combining ITA and CRTA

into ITA^+

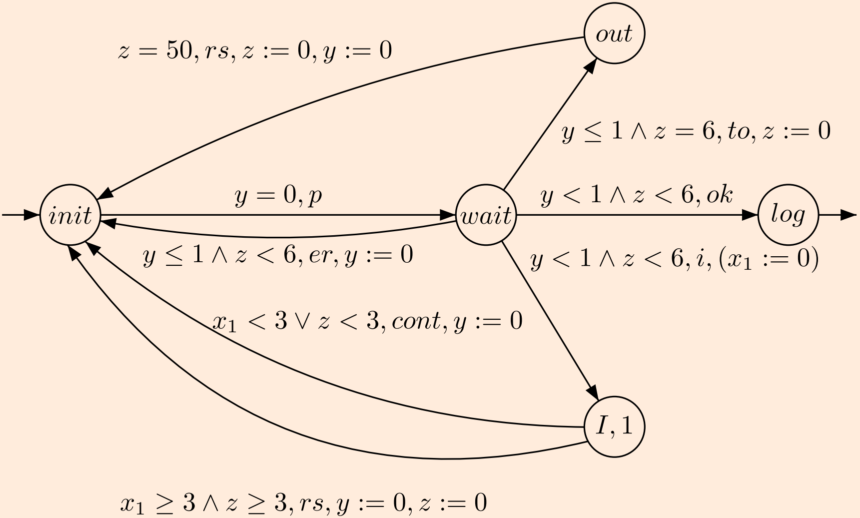
- A set Q of states with either a color or a level, and a velocity.
- A set X of interrupt clocks and a set Y of clocks with the features of CRTA clocks: color, lower and upper bound.
  Clocks with the color of the state are active in this state with same velocity.
  Exactly one interrupt clock active in states with a level.
- guards are of the form $\varphi_1 \wedge \varphi_2$, with φ_1 a guard on X and φ_2 a guard on Y, with the constraints of their respective models,
- updates are of the form $u_1 \wedge u_2$, with u_1 an update on X and u_2 an update on Y, also with the constraints of their respective models.

Reachability

The reachability problem remains decidable in the class ITA^+.
It belongs to 2-NEXPTIME (NEXPTIME with ITA_-^+) and is PSPACE-complete when the number of interrupt clocks is fixed.

An example of ITA$^+$

A login procedure



Outline

Conclusion and perspectives

Summary of results

- An appropriate model for a frequent pattern of discrete-event systems.
- Decidability of the reachability problem and contrasting complexity results.
- Incomparability with TA motivating a “decidable” combination of models.

Perspectives

- Lower bounds for the reachability problem.
- Model-checking ITA.