Qualitative Reasoning Based on Symbolic Probabilistic Evaluation: an Application

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Abstract

In the context of the fault models of system to be diagnosed, a new criterion is proposed to select the set of explicative hypotheses, which is less restrictive with respect to the *irredundancy condition*, that is to the request for minimal coverings (with respect to set inclusion). By means of examples involving an actual expert system for the diagnosis of dampness in building made of masonry, we stress that *irredundant* explanations are not always the most reliable ones.

Resumé

On souhaite modéliser les défaillances d'un système en vue de les diagnostiquer. Nous proposons une nouvelle méthode pour sélectionner l'ensemble des hypothè ses explicatives, qui est moins restrictive que la condition de non-redondance (c'est-à-dire l'exigence d'un recouvrement minimal au sens de l'inclusion ensembliste). Au moyen d'exemples pris dans un système expert pour le diagnostic de l'humidit é dans les bâtiments anciens, nous mettons en évidence que les explications non-redondantes ne sont pas toujours les plus fiables.

1 Introduction

In several formal approaches in which is adopted a fault model of the systeplicative hypotheses taken into account has to satisfy primarily two condition [8]:

- *covering condition*: the condition of explaining all of the manifestations that are present;
- *parsimony condition*: the condition of minimising the complexity of the explanation.

The last is usually interpreted as the search of the smallest covering of the present manifestations, with respect to the inclusion relationship (*irredundancy condition*).

In the formal approaches in which is adopted the correct behaviour model of the system to be diagnosed, the *covering condition* is substituted by the *consistency condition* , that is, the hypotheses on the abnormal behaviour of some components (abnormality assumptions) of the system have to be consistent both with the observations and with all the ot her components acting correctly. Even in this kind of approach, the final diagnosis has to satisfy the *parsimony condition* [10][4]. There exist several attempts of integrating fault models and correct behaviour models in which the *parsimony condition* plays a primary role [3].[1]

In the following sections, an alternative criterion for hypothesis selection is reported. In this approach, the *irredundancy condition* does not play a primary role in singling out the explicative hypotheses. In particular, it is not correct discard a priori the redundant hypotheses, especially when information useful to define a hierarchy between hypotheses is present. In general, *irredundancy condition* is not a reasonable criterion to adopt for bounding the number of possible hypotheses and, therefore, the complexity associated to the choice of the most plausible diagnosis. *Irredundancy condition* has to be considered as one of the factors to take eventually into account to establish a hierarchy between the possible diagnoses, and not necessary as the most important condition.

Following these considerations, a formal characterisation of the notion of diagnostic problem is proposed (section 3). In this formalisation, the *irredundancy condition* does not have a privileged position in the choice of the diagnosis.

Finally, an expert system based on this new formalisation is presented in section 5.

2 The B.A.T.J. model of abduction

The formal model proposed by Bylander *et al.* [2] is adequate for describing main logical-qualitative theories on diagnostic reasoning in which the condition of parsimony plays the primary role. In this model, an *abduction problem* is a tuple $\langle D_{\text{all}}, H_{\text{all}}, e, pl \rangle$, where:

- D_{all} is a finite set of all data to be explained. D stands for a set of data ($D \subseteq D_{\text{all}}$);
- H_{all} is a finite set of all the individual hypotheses. H stands for a set of individual hypotheses ($H \subseteq H_{\text{all}}$);
- e is a map from subsets of H_{all} $(\wp(H_{\text{all}}))$ to subsets of D_{all} $(\wp(D_{\text{all}}))$ (H explains $e(H)$);
- pl is a map from subsets of H_{all} $(\wp(H_{\text{all}}))$ to a partially ordered set (H as plausibility $pl(H)$).

"H explains $e(H)$" can be interpreted either as a logical implication "$H \to e(H)$", or as a causal implication "H can cause $e(H)$", etc. Therefore, it is not specified on

[1] A wide range of formal diagnosis model do not adopt the *parsimony condition* in order to select the explicative hypotheses, for instance those based on generalizations of traditional bayesian classification [6][7][8].

which kind of diagnostic inference is based the selection of a set H from the observations D.

$e(H)$ has to be interpreted as the set of observations explained by H, supposing not verified or irrelevant the hypotheses belonging to $H_{all}–H$ (H is a composite hypothesis).

$pl(H)$ could be a probability, a measure of belief, a fuzzy value, a degree of fit, or a symbolic likelihood. The only requirement is that the range of pl is partially ordered.

For any *abductive problem* $\langle D_{all}, H_{all}, e, pl \rangle$:

- H is *complete* if $e(H) = D_{all}$. That is, H explains all the data;
- H is *parsimonious* if $\neg\exists H' \subset H : e(H) \subseteq e(H')$ .. That is, no proper subset of H explains all the data that H does;
- H is an *explanation* if it is complete and parsimonious;
- H is a *best explanation* if it is an explanation, and if there is no explanation H' such that $pl(H') > pl(H)$.

Formally, an abduction problem is *monotonic* if:

$$\forall H, H' \subseteq H_{all} \left(H \subseteq H' \rightarrow e(H) \subseteq e(H') \right).$$

That is, any extension of the composite hypothesis H, namely H', explains all the data explained by H.

If a problem is monotonic, it can exist an hypothesis H such that $\bigcup_{h \in H} e(h) \subset e(H)$, that is, a composite hypothesis can explain additional data that are not explained by any of its elements. In the following, we refer at this sort of hypothesis interaction as "additive interaction".

A abductive problem is not monotonic if there is incompatibility between hypotheses. An *incompatibility abduction problem* is a tuple $\langle D_{all}, H_{all}, e, pl, \Im \rangle$ in which the new element ¡ is a set of subsets of H_{all} , indicating pairs or group of hypotheses that are incompatible with each other. For an incompatible problem:

$$\forall H \subseteq H_{all} \left((\exists I \in \Im \, (I \subseteq H)) \rightarrow e(H) = \varnothing \right).$$

Another interaction not allowed in monotonic abduction problems is *cancellation* .. In the following sections, this kind of interaction will be not take into account and we focus only on monotonic problems (except for incompatibilities).

3 An alternative criterion for hypothesis selection

The structure of solutions (*explanations*) required by B.A.T.J. model does not allow to uniquely define which hypotheses explain the single datum d_i .. For instance, let consider the following abductive problem $\langle D_{all}, H_{all}, e, pl \rangle$ in a medical diagnostic domain, where: $D_{all} = \{d_1, d_2, d_3\}$, $H_{all} = \{h_1, h_2\}$, $e(\{h_1\}) = \{d_1, d_3\}$,

$e(\{h_2\}) = \{d_2, d_3\}$, $e(\{h_1, h_2\}) = \{d_1, d_2, d_3\}$, and pl arbitrarily fixed. In this case, D_{all} represents the set of manifestations to be explained, H_{all} is the set of possible diseases, $e(h_i)$ is the set of all the manifestations of h_i. The solely possible explanation for D_{all} is $\{h_1, h_2\}$, but this explanation could have three different interpretations with respect to d_3: first, h_1 is mainly responsible for d_3; second, h_2 is mainly responsible for d_3; third, h_1 and h_2 are equally responsible for d_3.

To deal with this situation, we need to change both the explanation structure and the criterion of plausibility. The explanation structure has to be modified in order to make unique the interpretation, and the criterion of plausibility has to allow the int roduction of a hierarchy between the different interpretations of an explanation. For instance, setting $D_{\text{all}} = \{d_1, d_2, \ldots, d_k\}$, an explanation H could be accompanied by a tupla $P = (P_1, P_2, \ldots, P_k)$, with $H = \cup_{i=1}^{k} P_i$, in order to uniquely define, for each d_i , the subset P_i of H responsible for the presence of d_i; furthermore, the function pl could be defined onto the set of tuple $(P_1, P_2, \ldots, P_k)$, and $\forall i \in \{1, 2, \ldots, k\}\ P_i \neq \varnothing$ in order to satisfy the covering condition [5].

In the following, only diagnostic problems in which is requested the *covering condition* are considered. Furthermore, the role played by *parsimony condition* in diagnostic problems is analysed.

A parsimony condition more inclusive than that proposed by the B.A.T.J. model, is the following: setting $D_{\text{all}} = \{d_1, d_2, \ldots, d_k\}$, a complete hypothesis H, for which has been defined the tupla $(P_1, P_2, \ldots, P_k)$, is an explanation if and only if:

$$\forall i \in \{1, 2, \ldots k\}\ d_i \in e(P_i)\ \wedge\ \neg\exists H' \subset P_i : d_i \in e(H')\ \wedge\ e(H) = D_{\text{all}} \qquad (1).$$

In this case, one selects between the set of complete hypotheses only those for which P_i is composed of individual hypothesis or of a set of individual hypotheses with additive interaction.

Condition (1) is justified when the purpose is that of eliciting the principal hypothesis for each manifestation. If H is *complete* and *parsimonious* then there exists a partition $(P_1, P_2, \ldots, P_k)$ of H which satisfies the c ondition (1). While, if the tupla $(P_1, P_2, \ldots, P_k)$ satisfies the condition (1), the set $H = \cup_{i=1}^{k} P_i$ could not be *parsimonious* .. Thus, the condition (1) is weaker than the condition of parsimony proposed by B.A.T.J.

An actual example in the field of architecture can help clarifying the last statement. The domain is that of dampness diagnosis in building made of masonry. Let $D_{\text{all}} = \{d_1, d_2\}$ be the set of manifestations to be explained, where:

- d_1 continuous efflorescence in the basement and ground floor;
- d_2 erosion of external walls in the basement and ground floor;

$H_{\text{all}} = \{h_1, h_2, \ldots, h_n\}$ the set of all the possible causes where, in particular:

- h_1 water from bearing stratum;
- h_2 private water pipe breaking;

and: $e(\{h_1\}) = \{d_1, d_2\}$, $e(\{h_2\}) = \{d_2\}$, $e(\{h_1, h_2\}) = \{d_1, d_2\}$.

The complete hypotheses $\{h_1\}$ and $\{h_1, h_2\}$ are selected applying condition (1). To these hypotheses are associated respectively the tuple $P' = (\{h_1\}, \{h_1\})$ and $P'' = (\{h_1\}, \{h_2\})$. As a matter of fact, h_1 can cause both d_1 and d_2, while h_2 can cause only d_2. On the contrary, applying the parsimony condition of the B.A.T.J. model we obtain only one explanation for D_{all}, i.e. $\{h_1\}$, while the hypotheses $\{h_1, h_2\}$ will be discarded. The hypothesis h_2 is not taken into account because it is not present in any explanation. In this situation, the hypothesis h_2 is rejected (parsimony requirement) without checking whether h_2 has a level of plausibility higher than h_1 with respect to d_2, i.e. $pl(\{h_1, h_2\}) > pl(\{h_1\})$.

This example belongs to those diagnostic problems in which the most plausible solutions are not necessary *irredundant*.

The parsimony condition is a plausible condition as many others, and the problem of finding the best explanation introduced by the B.A.T.J. model forces the *parsimony condition* to be applied before of any other selection criterion.

4 Plausible and complete solutions *versus* parsimonious and complete solutions

The examples introduced in the previous section suggest to modify the definitions of *explanation* and *best explanation*.

Using the same terminology, we introduce two differences with respect to the B.A.T.J. model, that is: the abductive problem will be represented by the tuple $\langle D_{\text{all}}, H_{\text{all}}, e, pl_{\text{rel}} \rangle$, in which e is represented by the couple $\langle C, \Im \rangle$, and pl_{rel} presents a structural difference with respect to pl.

The couple $\langle C, \Im \rangle$ is formed by ¡, the set of subsets of H_{all}, indicating pairs or group of hypotheses that are incompatible with each other, and by the relation C defined as follow: $d \in D_{\text{all}}$, $H \in \wp(H_{\text{all}})/\{\varnothing\}$, $\langle d, H \rangle \in C$ if and only if H is a single hypothesis which may cause d, or H is a composite hypothesis which may cause d by means of the contemporaneous presence of all its elements (addictive interaction). The relationship between C and e is:

$$\langle d, H \rangle \in C \quad \text{if and only if} \quad [d \in e(H)] \wedge [\neg \exists H' \subset H : d \in e(H')].$$

The relation C generalises that introduced by the Generalised Set Covering [9]. In fact, the GSC does not take into account both possible additive interaction and incompatibility between hypotheses.

The set of composite hypotheses responsible for the presence of the manifestation d it will be denoted by $S(d) = \{H : \langle d, H\rangle \in C\}$.

If $\Pi = \langle D_{\text{all}}, H_{\text{all}}, e, pl_{\text{rel}}\rangle$ is an abductive problem, with $D_{\text{all}} = \{d_1, d_2, \ldots, d_k\}$, then the set of tuple $(P_1, P_2, \ldots, P_k)$ belonging to $S(d_1) \times S(d_2) \times \ldots \times S(d_k)$ will be denoted by $S(\Pi)$. To each manifestation d_i is associated the explanation P_i. The function pl_{rel} could be a probability, a measure of belief, etc., as for the function *pl* defined by the B.A.T.J. model, but the domain is $S(\Pi)$.

For any abductive problem $\Pi = \langle D_{\text{all}}, H_{\text{all}}, e, pl_{\text{rel}}\rangle$:

- the tupla $P = (P_1, P_2, \ldots, P_k)$, with $H = \cup_{i=1}^{k} P_i$, is a *solution* for Π if $P \in S(\Pi) \wedge e(H) = D_{\text{all}}$. The interpretation of the tupla $(P_1, P_2, \ldots, P_k)$ is unique because it associates to the datum d_i the explanation P_i;
- the tupla $P = (P_1, P_2, \ldots, P_k)$ is an *irredundant solution* if:
$$\neg\exists P' = (P_1', P_2', \ldots, P_k') : \cup_{i=1}^{k} P_i' \subset \cup_{i=1}^{k} P_i;$$
- the tupla $P = (P_1, P_2, \ldots, P_k)$ is a *best solution* for Π if it is a *solution* for Π and $\neg\exists P' : pl_{\text{rel}}(P') > pl_{\text{rel}}(P)$, that is, there are no other solutions P' with plausibility greater than P (*maximum plausibility condition*).

Adopting this char acterisation, the irredundancy condition for the complete solutions of abductive problems has no more priority. The request is only that the explanation for each manifestation has to be composed either by a single hypothesis or by a set of hypotheses with additive interaction. Thus, to obtain the set of *best solutions* we the *maximum plausibility condition* is applied. The parsimony condition of the B.A.T.J. model can be eventually included in defining the function pl_{rel} , i.e. it is possible but not necessary to consider as well the irredundancy of the solutions to establish a plausibility hierarchy between them. The irredundancy condition is only one of the possible constraints that a *best solution* has to satisfy, and it has not necessary to be present, or better, it has not always to be the most important condition [5]. An actual example of this situation is given in the next section.

5 An expert system for the diagnosis of dampness in buildings made of masonry

The diagnostic problem the expert system deal with can be formalised by an instance $\langle D_{\text{all}}, H_{\text{all}}, e, pl_{\text{rel}}\rangle$ of the *best solution* searching problem; where the set $D_{\text{all}} = \{d_1, d_2, \ldots, d_k\}$ represents the set of all the manifestations of dampness wearout, and the set $H_{\text{all}} = \{h_1, h_2, \ldots, h_n\}$ represents the set of all the wearout causes. Due to the absence of incompatibility between hypotheses (monotonic problem), the set ¡ is empty. Furthermore, $\langle d,H\rangle \in C$ if and only if the expert system abduces H as possible explanation for d.

A set of symbolic values are used by the expert system in order to represent the correlation between explicative hypotheses and manifestations [1];[2] these values are dynamically calculated by the system with respect to the data input given by the user. The data concern with: building location, dampness location and their temporal evolution.

Setting $H \in S(d_i)$, the coefficients of correlation calculated by the system will be denoted by $corr(H,d_i)$.

The criterion of plausibility associated to the function pl_{rel} is defined as follow [5]: $\forall P,P' \in S(\Pi)$, with $P=(P_1,P_2,\ldots,P_k)$ and $P'=(P_1',P_2',\ldots,P_k')$, $pl_{\text{rel}}(P) > pl_{\text{rel}}(P')$ if and only if the following conditions are satisfied:

1. $\forall i \in \{1,2,\ldots,k\}\ corr(P_i,d_i) \geq corr(P_i',d_i)$;

2. if $\forall i \in \{1,2,\ldots,k\}\ corr(P_i,d_i) = corr(P_i',d_i)$ then it has to turn out $\cup_{i=1}^{k} P_i \subset \cup_{i=1}^{k} P_i'$;

and the function pl_{rel} can be defined as:

$$\forall P=(P_1,P_2,\ldots,P_k) \in S(\Pi)\ \ pl_{\text{rel}}(P) = \left(\cup_{i=1}^{k} P_i, \left(corr(P_1,d_1),\ldots,corr(P_k,d_k)\right)\right)$$

that is, pl_{rel} is the couple formed by $\cup_{i=1}^{k} P_i$ and by the coefficients of correlation of the coordinates of H. The ordering associated with the range of pl_{rel} is implicitly defined by conditions 1. and 2. The value of $corr(P_i,d_i)$ indicates the plausibility associated to the hypothesis P_i with respect to manifestation d_i. More in general, the plausibility associated to an hypothesis H_i , with respect to a manifestation d_i, can depend upon the hypotheses chosen to explain the other manifestations.

Applying this criterion, a *solution* $P=(P_1,P_2,\ldots,P_k)$ has maximum plausibility only if it has maximal correlation with respect to all the manifestations of D_{all}, i.e. only if: $\forall i \in \{1,2,\ldots,k\}\ \ \neg\exists P_j' \in S(d_i) : corr(P_j',d_i) > corr(P_i,d_i)$. This is the first condition the *best solutions* have to satisfy and, however, it is only a necessary condition. The *best solutions*, chosen between those with maximum correlation, are solely the *solutions* more parsimonious (condition 2.).

The *best solutions* selected by the system are the most parsimonious *solutions* between those with maximum correlation, this means that the *solutions* chosen are not always *irredundant solutions* .. In fact, the parsimony condition is applied <u>after</u> the maximum correlation condition.

Reminding the example reported in section 3, independently from the definition of pl_{rel}, there not exist *best explanations* including the hypothesis h_2. Supposing that

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[2]The possible values for the coefficients of correlation are high, medium, low and null ordered as follow high>medium>low>null.

$corr(\{h_2, d_2\}) > corr(\{h_1, d_2\})$, the exclusion of h_2 is not justified. Considering, instead, the diagnosis as a problem of finding the *best solutions*, the *solution* $(\{h_1\},\{h_2\})$ is the *best solution*, in fact it has plausibility greater than the *solution* $(\{h_1\},\{h_1\})$.

6 Concluding remarks and future works

The formalisation proposed in section 4 has to be ulteriorly generalised in order to be applied to a wider set of diagnostic problems. In particular, it is not so fair to adopt the *covering condition* when the model is not complete, that is, when all the possible causes explaining the observations are not present in the model. In this case, an approach based on consistency [3] is much better. Furthe rmore, it is necessary to take into account the effects generated by the absence of expected manifestations to evaluate the plausibility of a solution, and the possibility of indirect causal associations between disorders and manifestations (causal chainin g).

The analysis of computational complexity for the problem of finding a *solution* or a *best solution* , is the aim of our future work. In particular, we want to analyse the effect on the computational complexity of these kind of problems when we adopt a selection criterion weaker than *irredundancy* .. (For a detailed analysis on the computational complexity of problems of finding an *explanation* or a *best explanation* the reader is referred to [2].) Starting from simplifying hypotheses analogous to those introduced in the B.A.T.J. model, the problem of finding a *solution* is not more complex than that related to an *explanation*.

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