Parameterization of Surfaces

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Outline

- Context
- Background
- Problem Statement
- Strategy
- Metric Distortion
- Conformal parameterization techniques
- Cone singularities
- Our algorithm
- Experiments
- Perspectives

Context

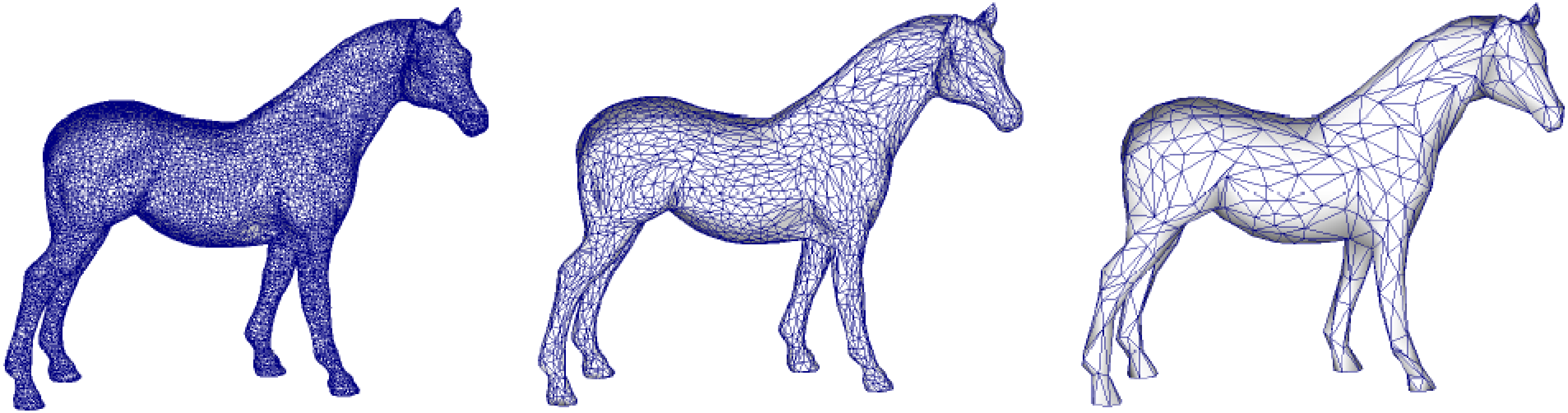
- Digital entertainment

[8]

[Nintendo]

Context

- Triangle mesh



[5]

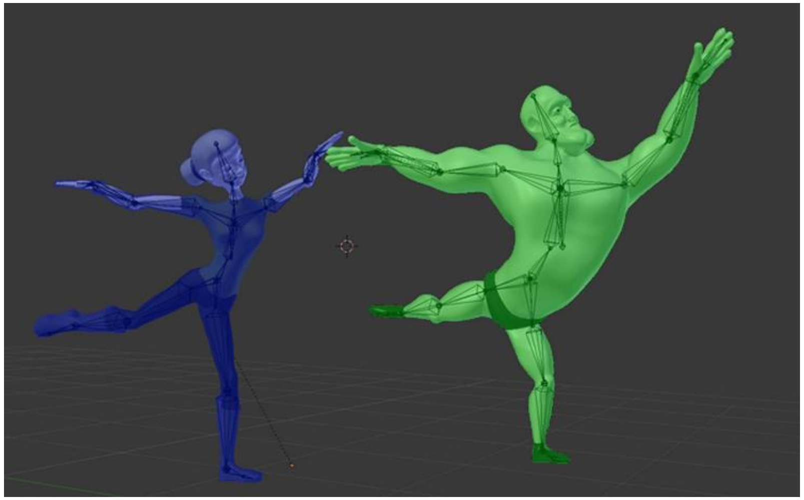
Context

- Meta-data
- Texture

[7]

Context

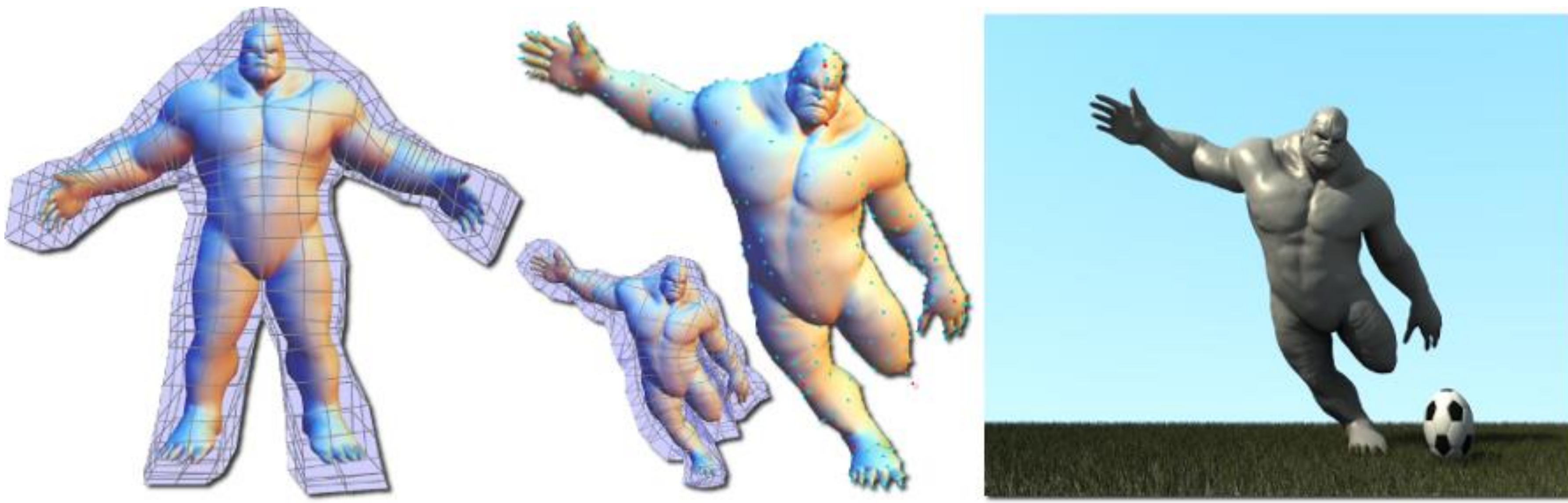
- Meta-data
- Animation skeleton



[14]

Context

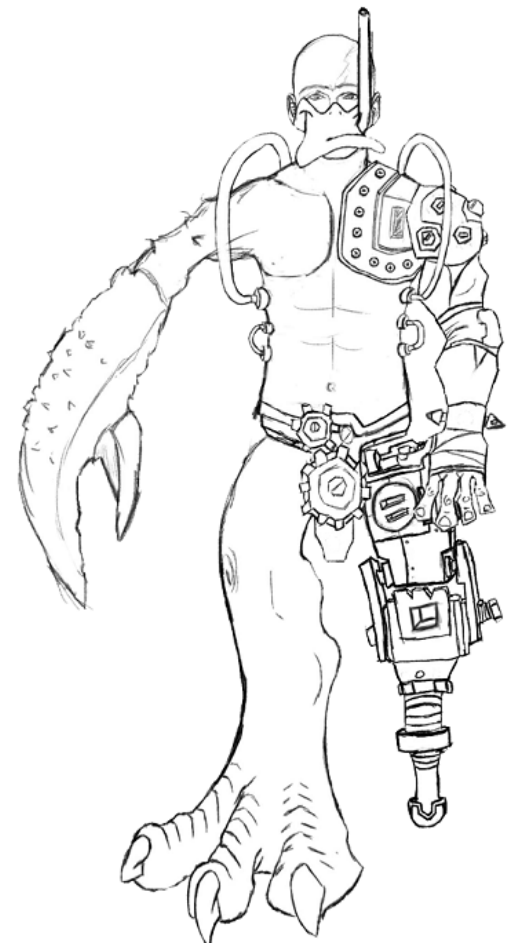
- Meta-data
- Deformation cages



[15]

Context

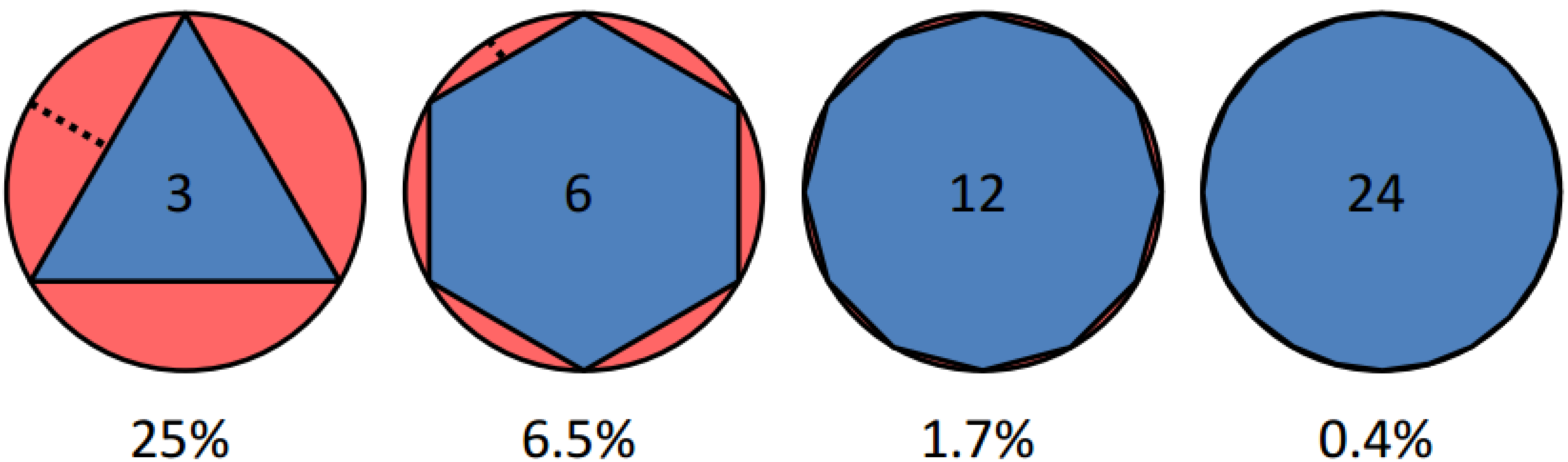
- Optimize the production and editing chain of 3D content



[CrABEx Project]

Background

- Definitions
  - Polygonal mesh



[3]

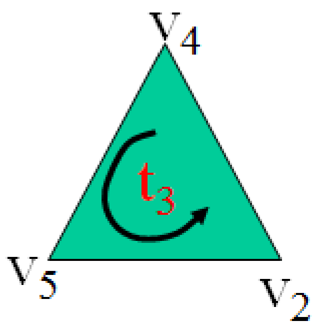
Background

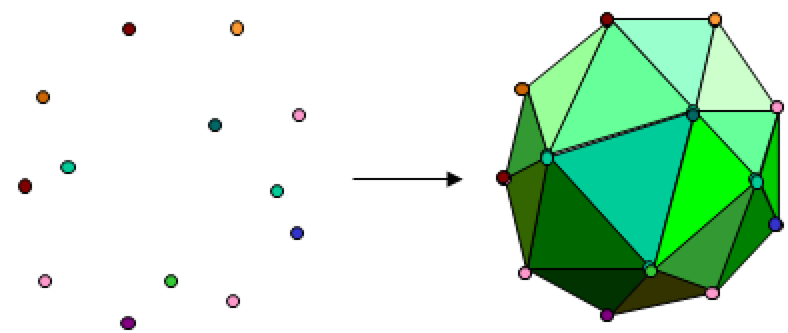
- Triangle mesh
  - Geometry

vertex 1	x	y	z
vertex 2	x	y	z
vertex 3	x	y	z

  - Connectivity

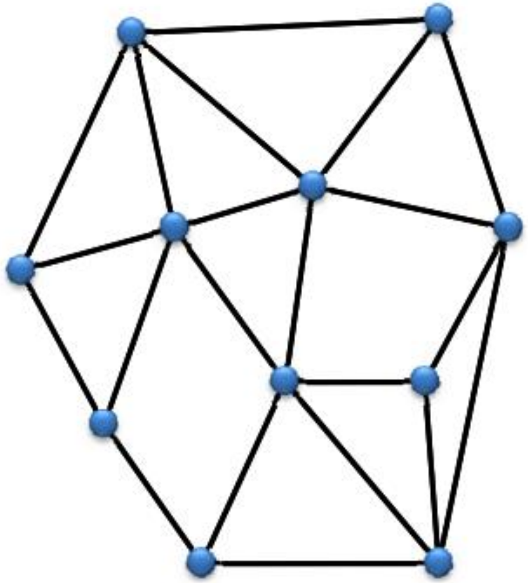
Triangle 1	1	2	3
Triangle 2	3	2	4
Triangle 3	4	5	2
Triangle 4	7	5	6
Triangle 5	6	5	8
Triangle 6	8	5	1



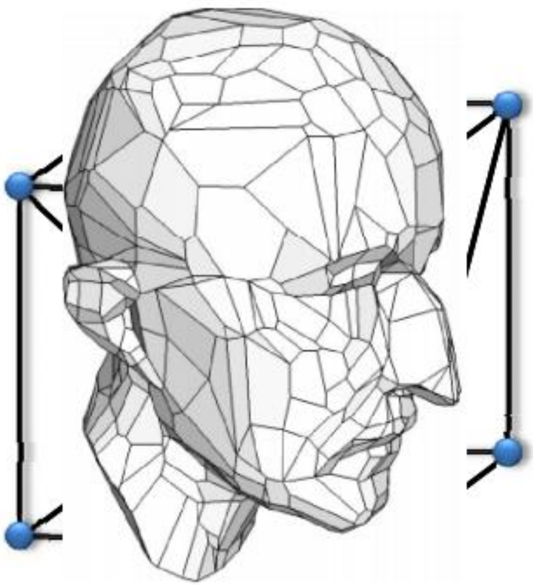


Background

- Triangle mesh
  - Connectivity
    - Seen as a graph
      - Graph embedding



Embedded in $\mathbb{R}^2$



Embedded in $\mathbb{R}^3$

[3]

Background

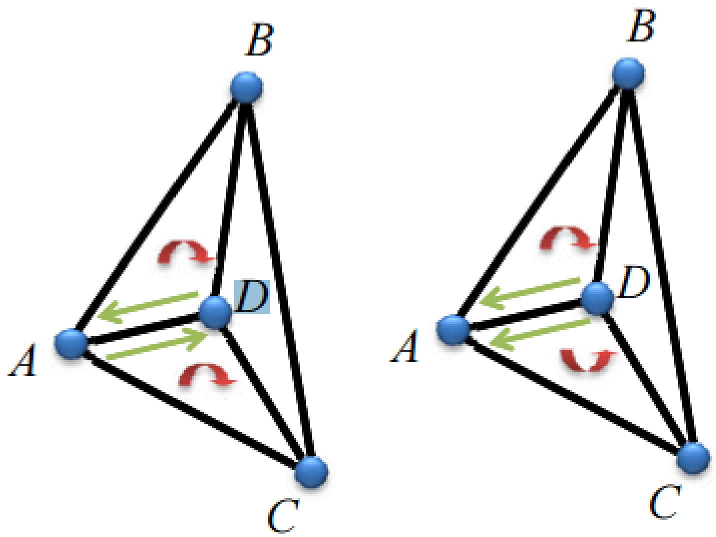
- Definitions
  - Topology
    - Genus

Genus 0 Genus 1 Genus 2

[3]

Background

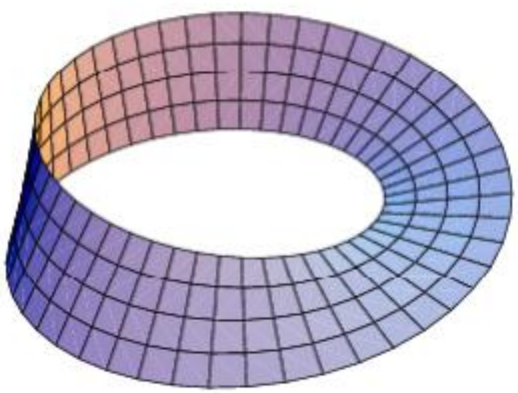
- Definitions
  - Orientability



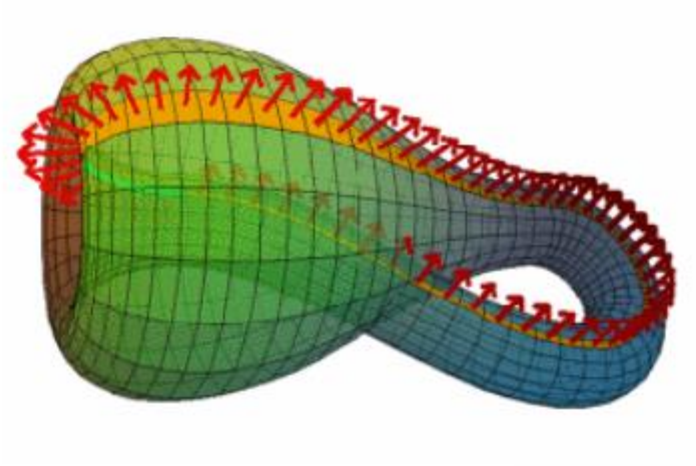
[3]

Background

- Definitions
  - Orientability



Mobius Strip

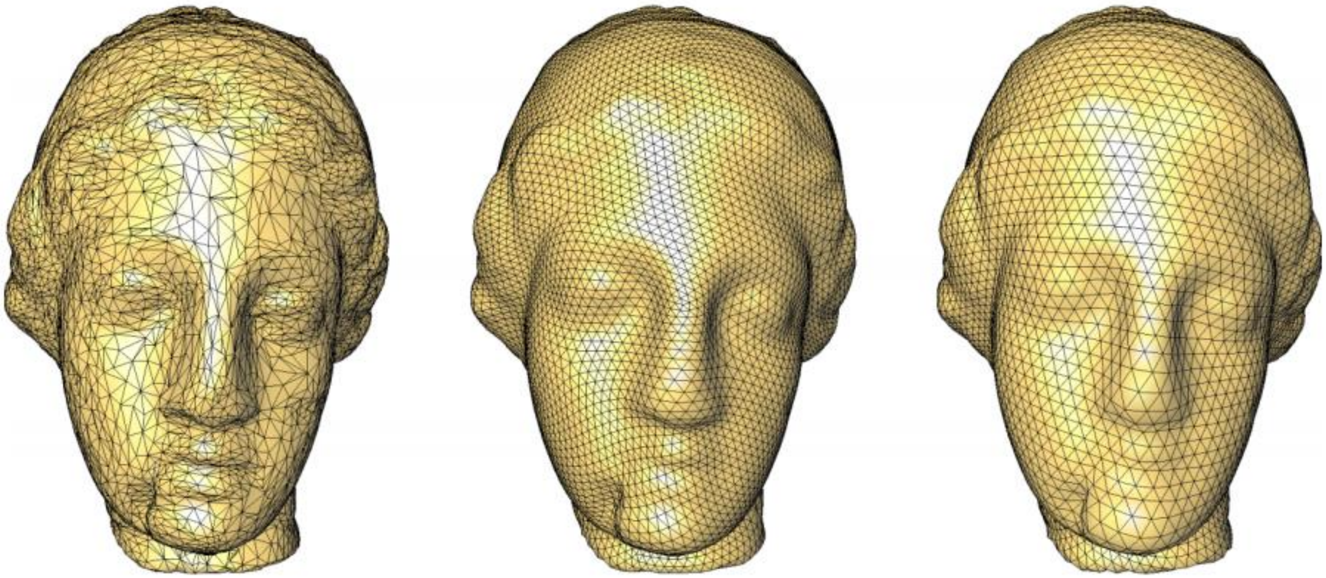


Klein Bottle

[3]

Background

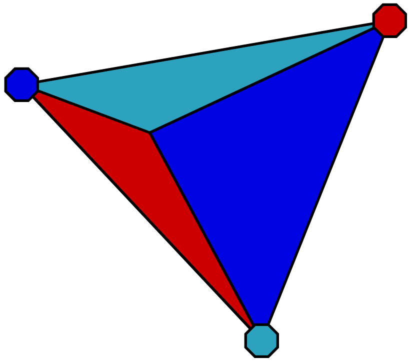
- Definitions
  - Simplicial complex
  - Triangulation



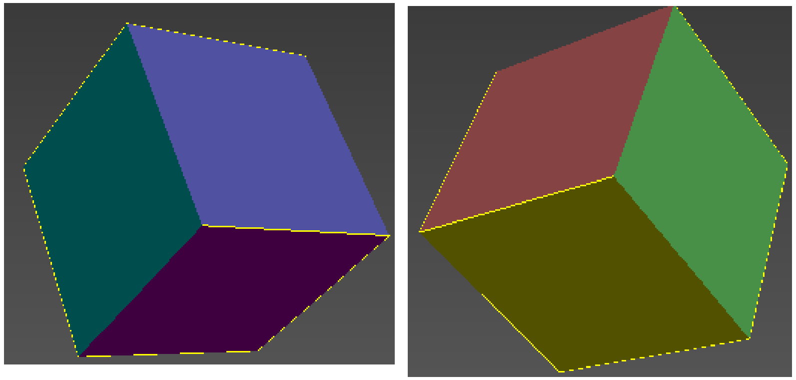
[1]

Background

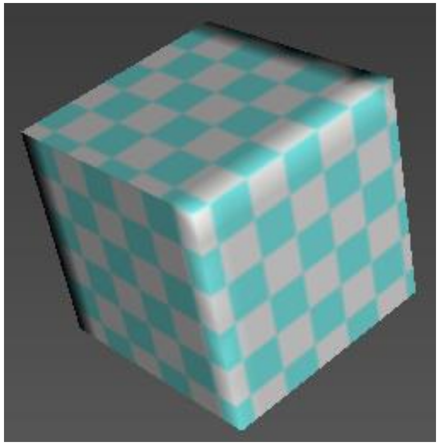
- Triangle mesh
  - Attributes
    - Color



    - Normals

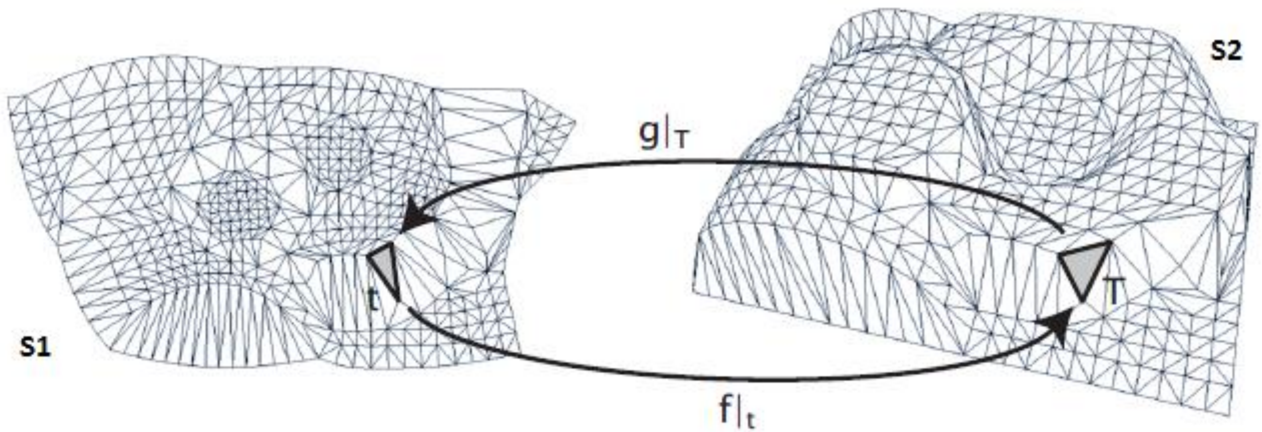


    - Texture



Problem Statement

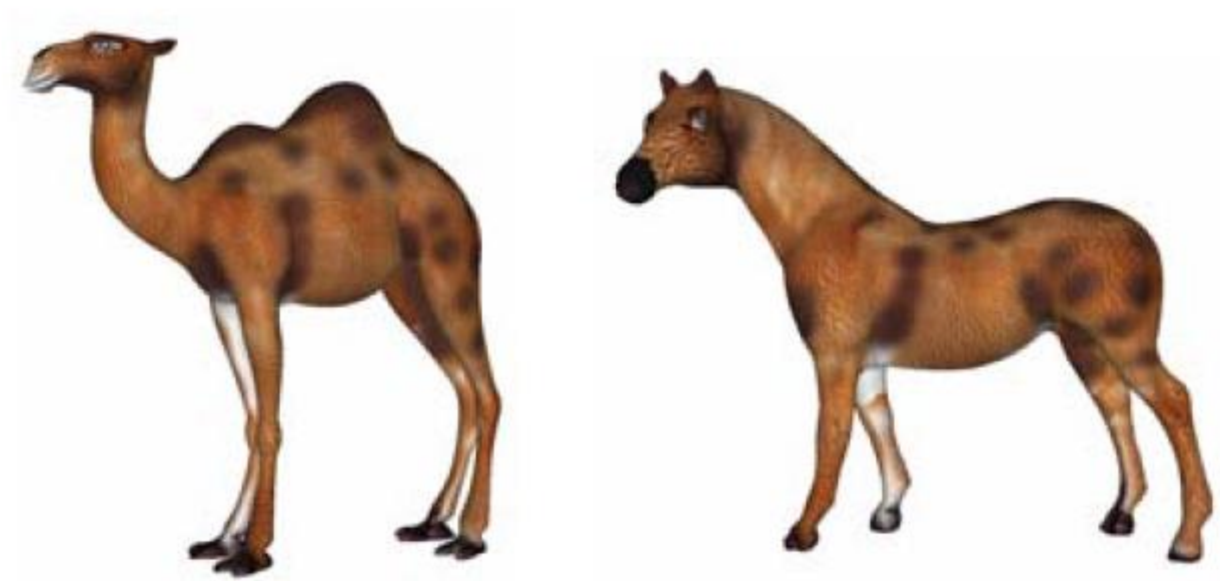
- Consistent bijective mapping



[5]

Problem Statement

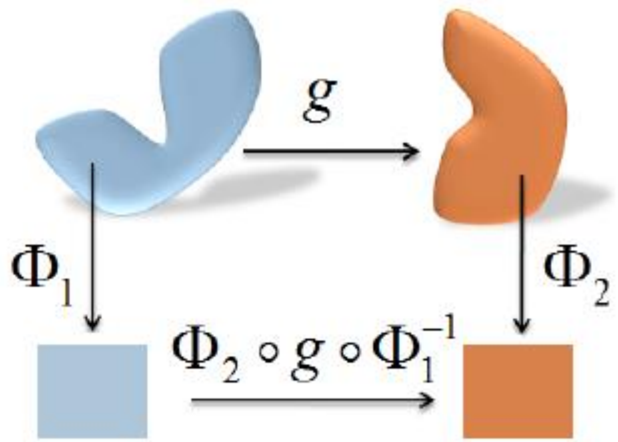
- Consistent bijective mapping



[6]

Strategy

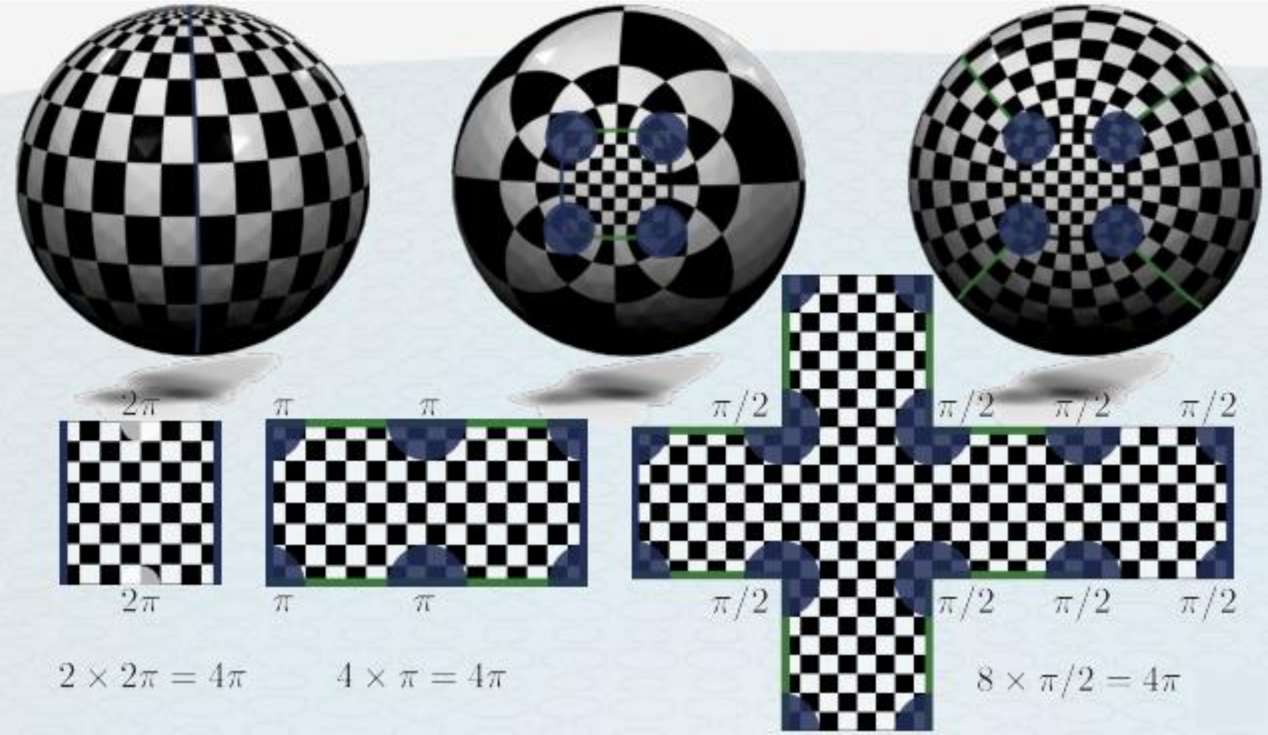
- Simplify
  - g, Φ_1, Φ_2 =?



[9]

Strategy

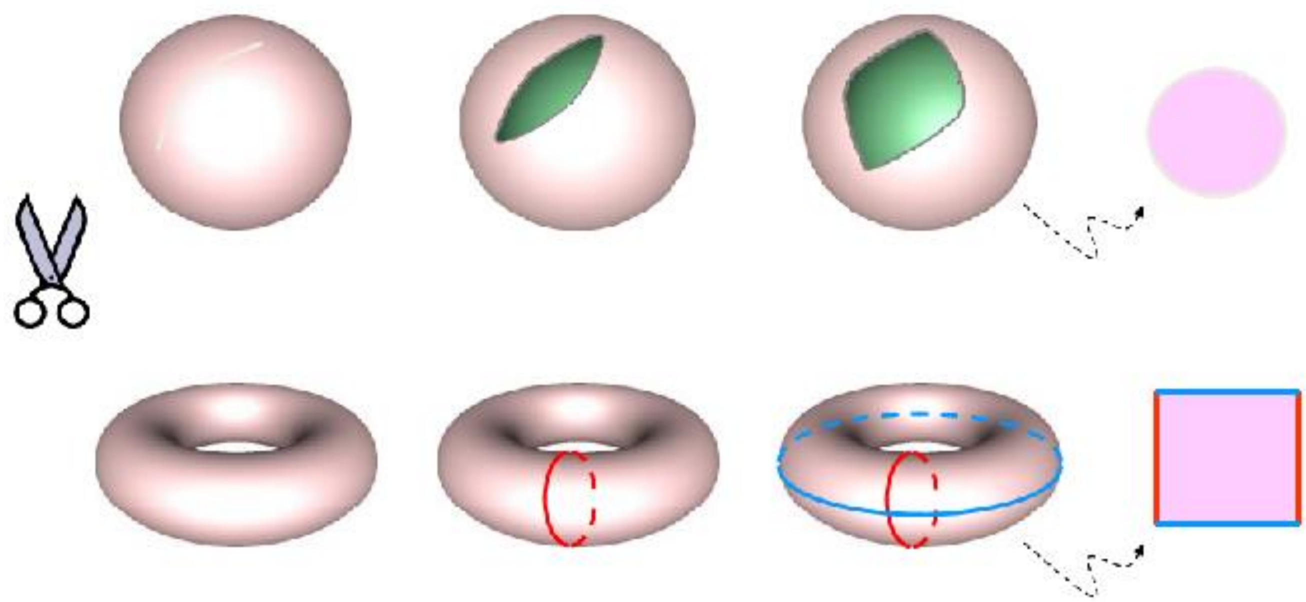
- Scientific issues
  - $g,\ \Phi_1,\ \Phi_2 = ?$
  - Generality
    - Diff param dom



[14]

Strategy

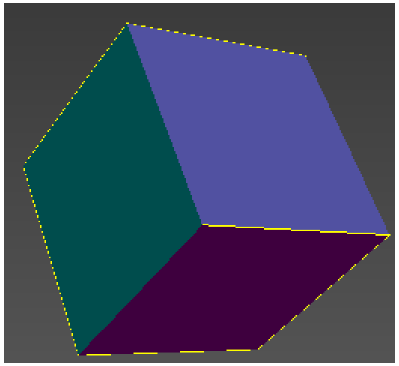
- Scientific issues
  - g, Φ_1, Φ_2 =?
  - Generality
    - Topology

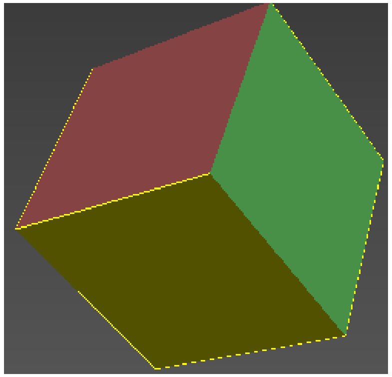


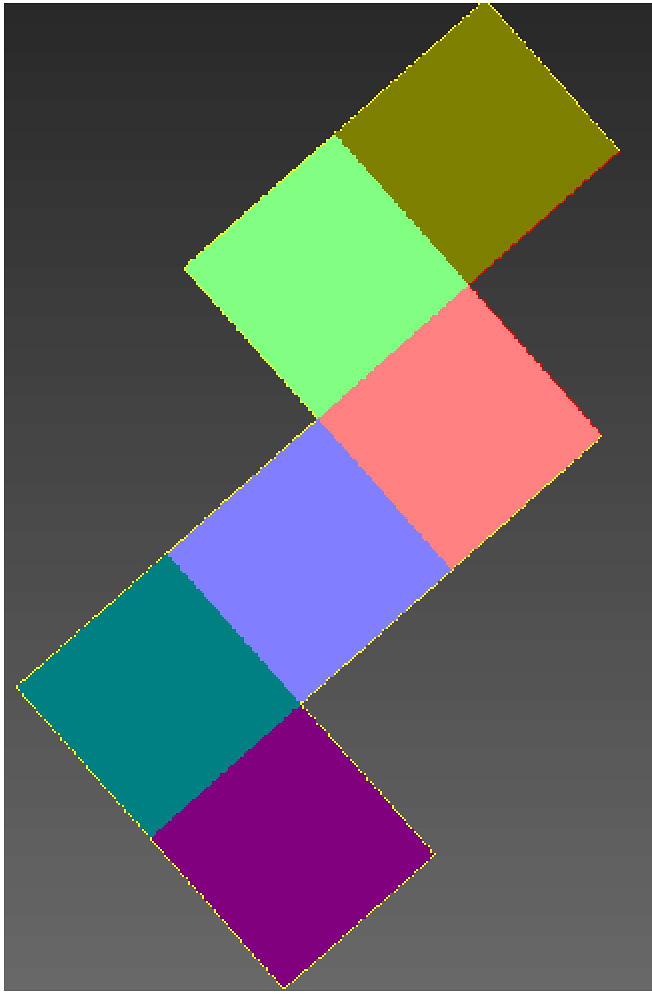
[3]

Strategy

- Scientific issues
  - g, Φ_1, Φ_2 =?
  - Generality
    - Topology



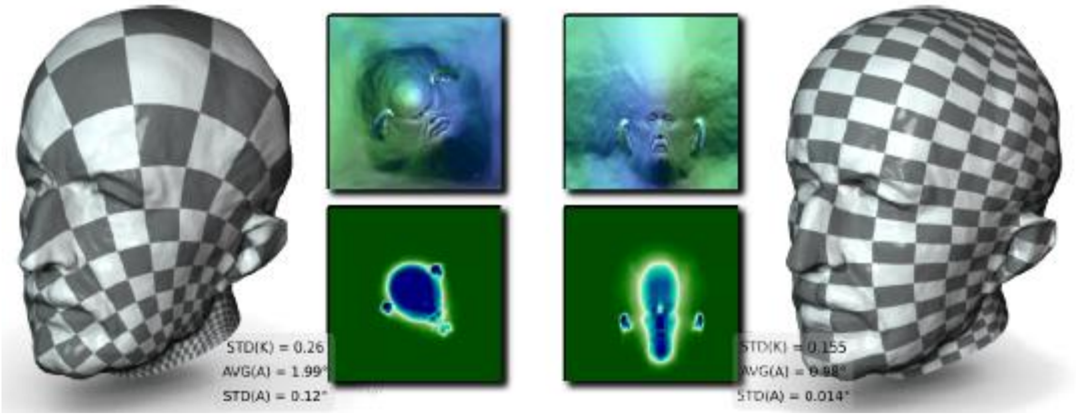




Strategy

- Scientific issues
  - g, Φ_1, Φ_2 =?
  - Generality
  - Accuracy
  - Timing

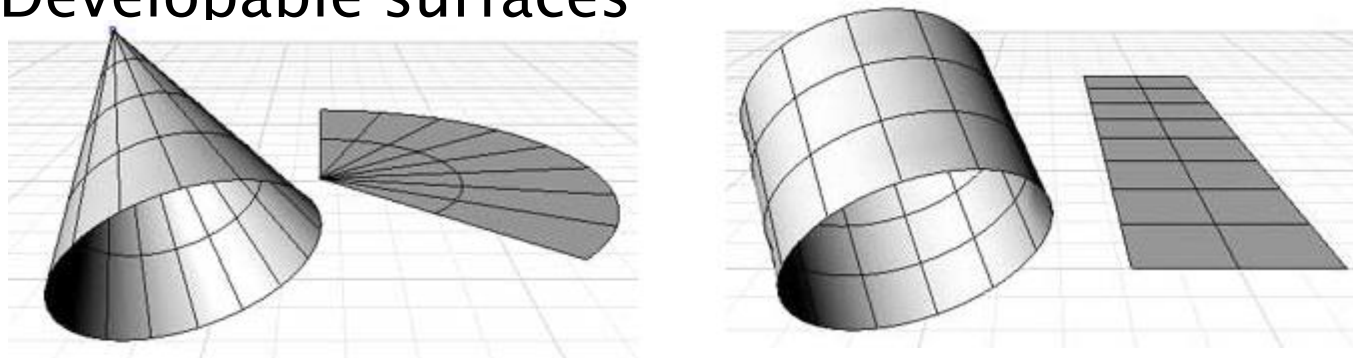
Accuracy



[13]

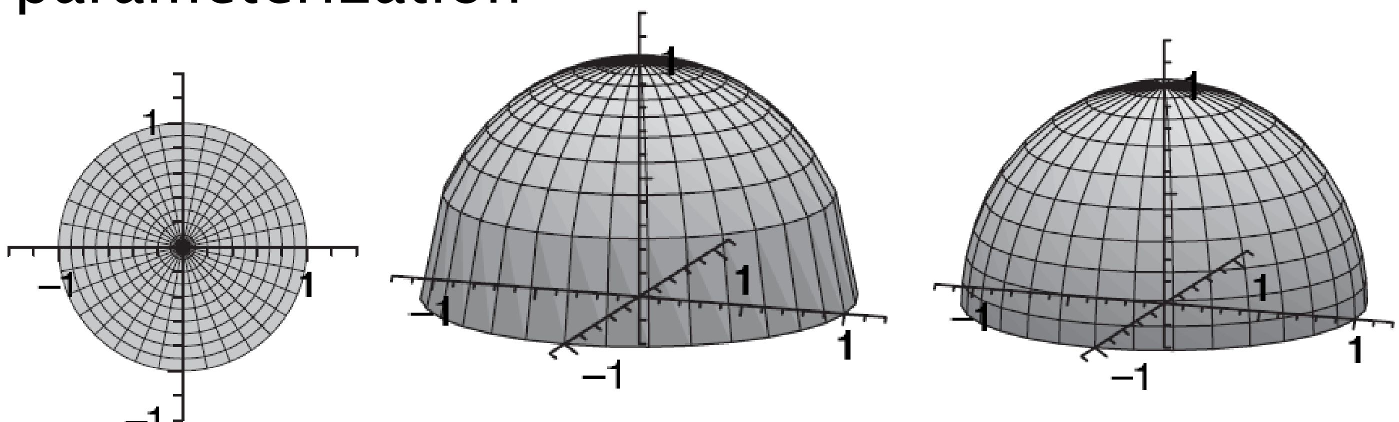
Metric Distortion

- Developable surfaces



[16]

- Non-unique parameterization

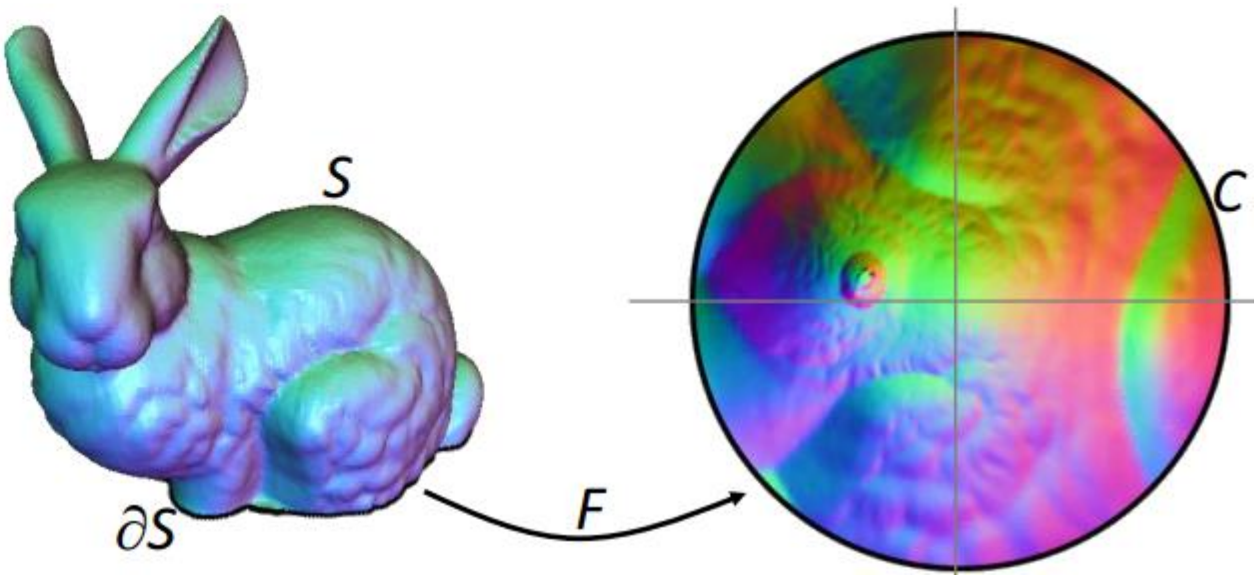


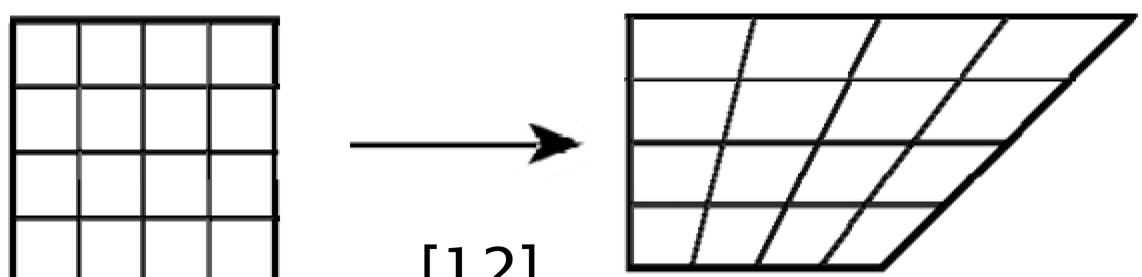
[5]

Metric Distortion

- Harmonic maps

$$\Delta u = 0, \Delta v = 0$$

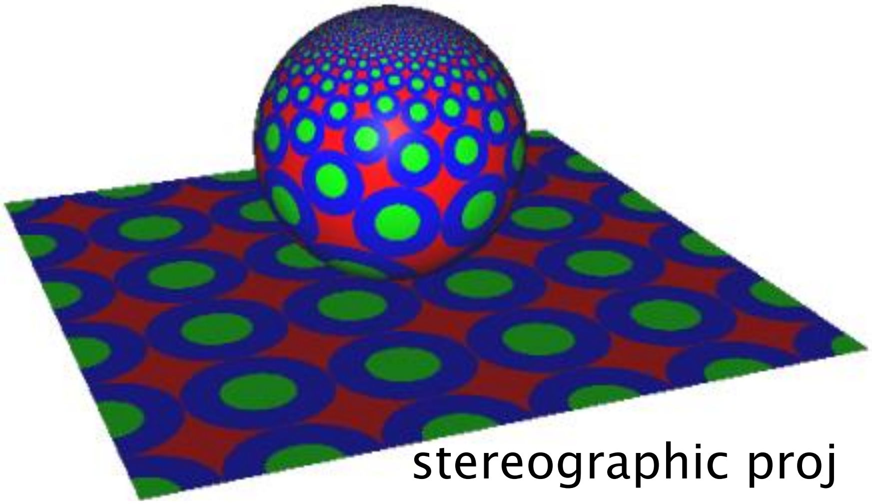




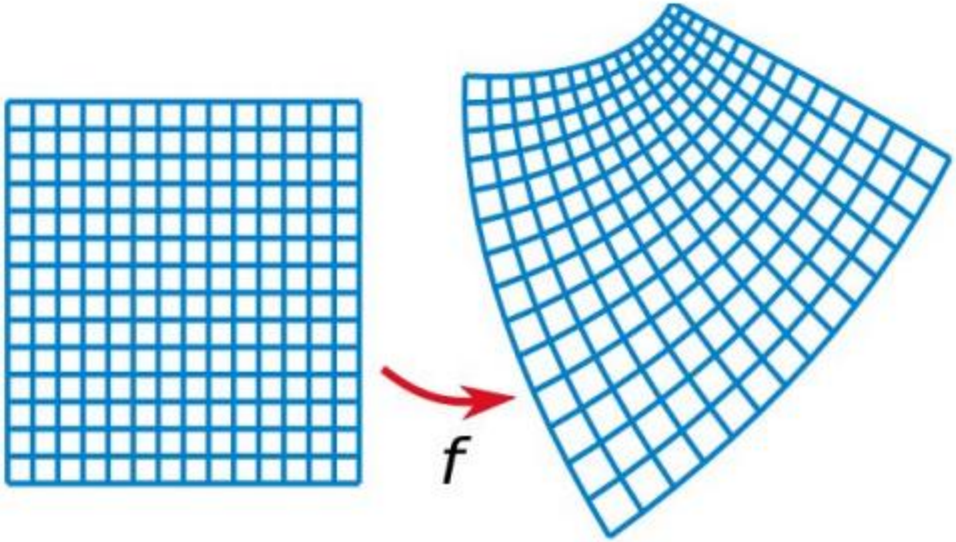
[12]

Metric Distortion

- Conformal maps
  - $||\nabla u|| = ||\nabla v||$,
  - $\nabla u * \nabla v = 0$



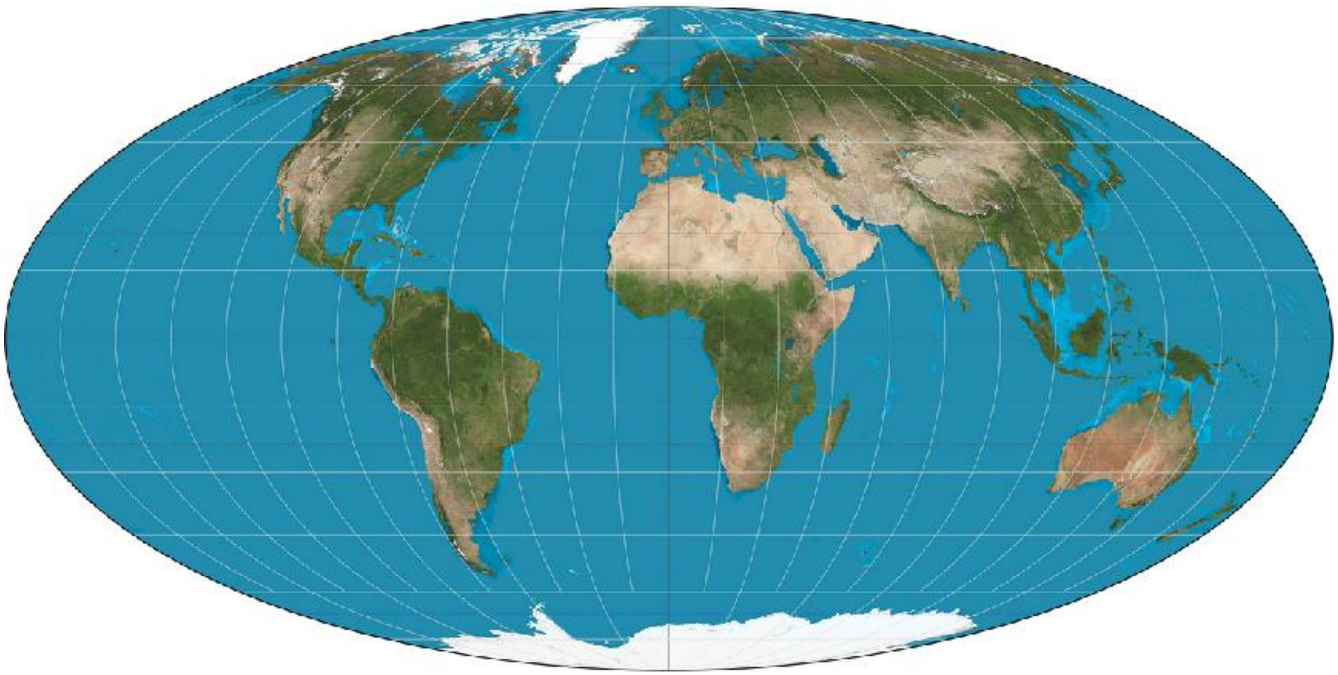
stereographic proj



[12]

Metric Distortion

- Equiareal maps

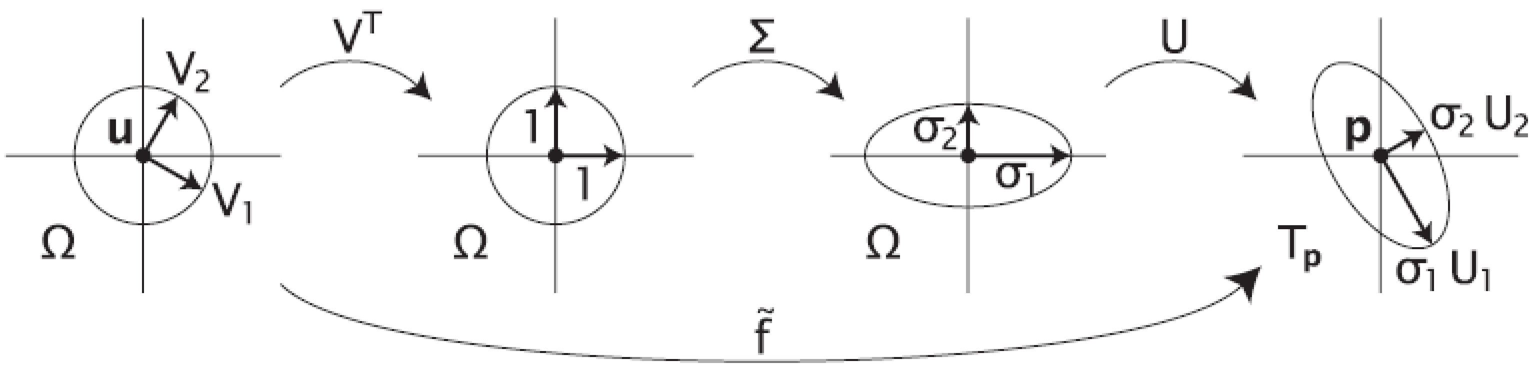


[17 The equal-area Mollweide projection]

Metric Distortion

- SVD decomposition of the map

$$J_f = U\Sigma V^T = U \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{pmatrix} V^T$$

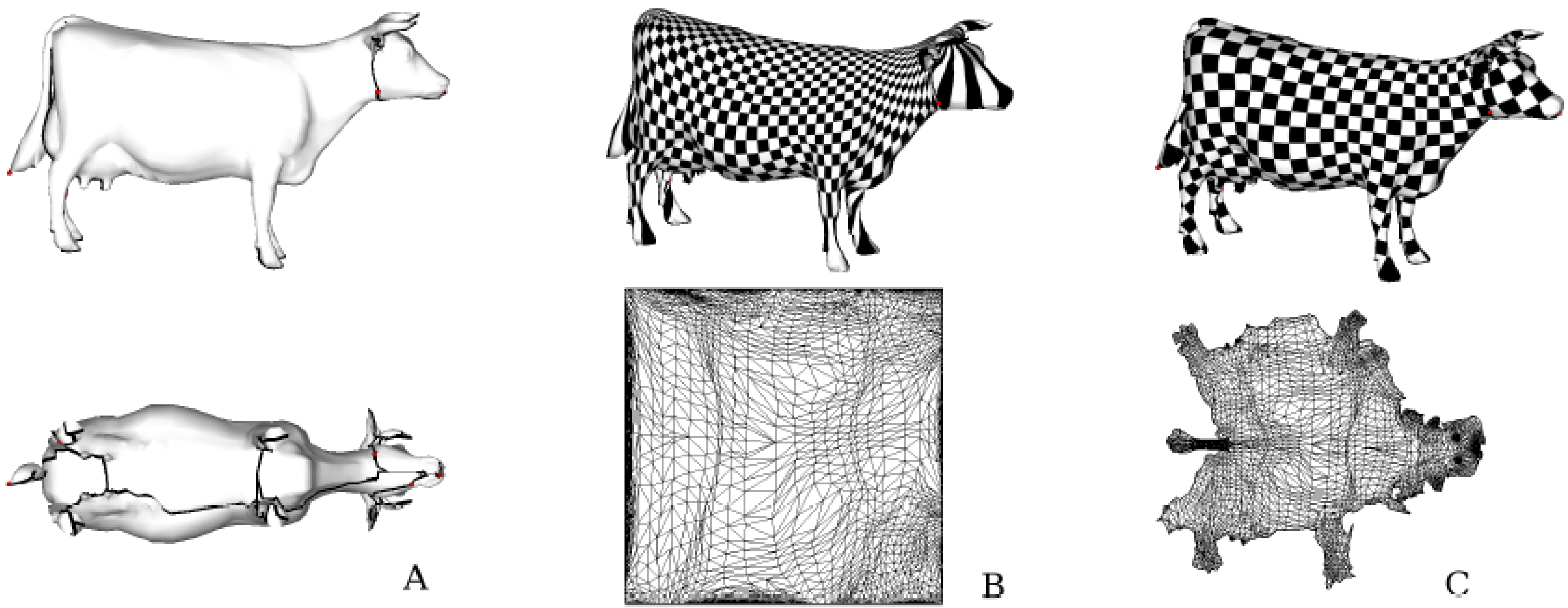


[5]

- As a consequence, any circle of radius r around u will be mapped to an ellipse with semi-axes of length rσ1 and rσ2 around p and the orthonormal frame [V1, V2] is mapped to the orthogonal frame [σ1U1, σ2U2].

Conformal Parameterization techniques

- Fixed boundary vs Free boundary



[5]

Conformal Parameterization techniques

- LSCM
  - Description:
    - Minimize the violation of Riemann's conditions in a least squares sense

$$\nabla v = \mathrm{rot}_{90}(\nabla u) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \nabla u$$

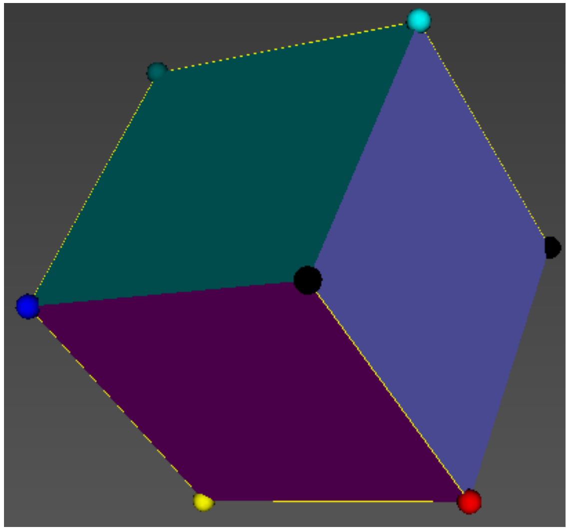
    - Minimize a distortion energy.

$$E_{LSCM} = \sum_{T=(i,j,k)} |T| \left\| \mathrm{M}_T \begin{pmatrix} v_i \\ v_j \\ v_k \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathrm{M}_T \begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix} \right\|^2$$

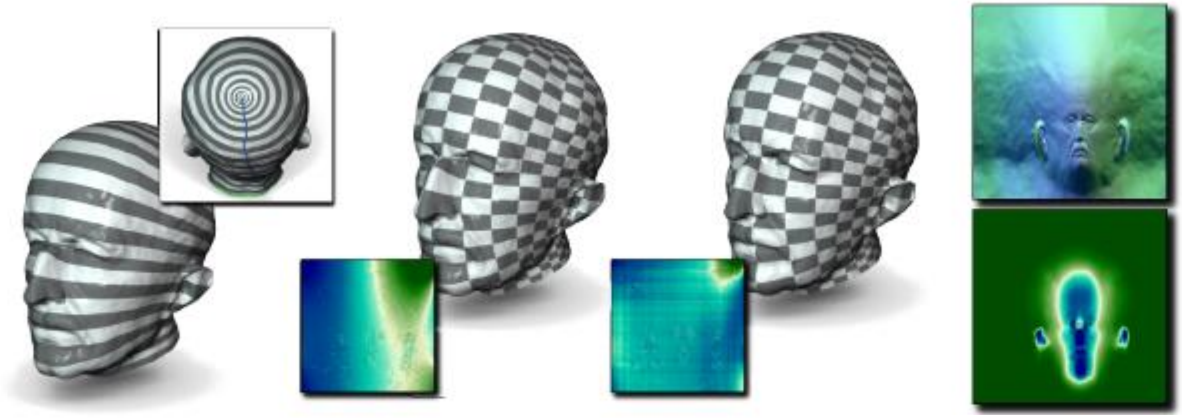
$$\begin{pmatrix} \partial u / \partial X \\ \partial u / \partial Y \end{pmatrix} = \mathrm{M}_T \begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix} = \frac{1}{2|T|_{X,Y}} \begin{pmatrix} Y_j - Y_k & Y_k - Y_i & Y_i - Y_j \\ X_k - X_j & X_i - X_k & X_j - X_i \end{pmatrix} \begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix}$$

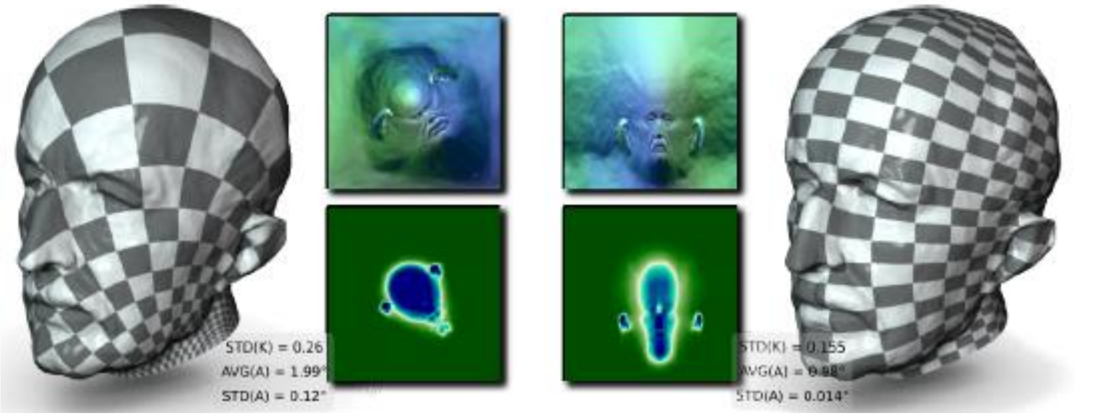
      - Combine the conformality condition and the linearity of the mapping (inside a triangle) in a least squares sense.

Cone singularities



- Absorb distortion
- Cut the mesh



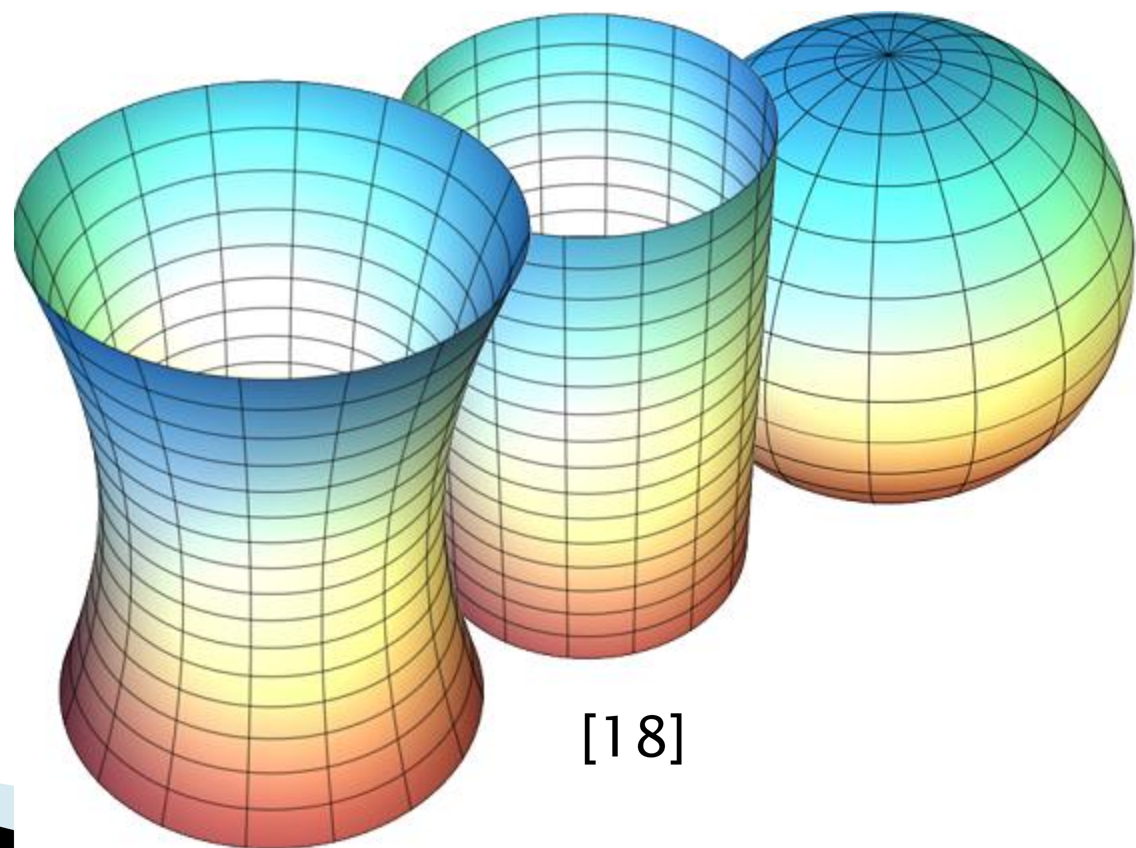


[13]

Cone singularities

- Gaussian curvature
- Angle deficit
- Gauss-Bonnet theorem

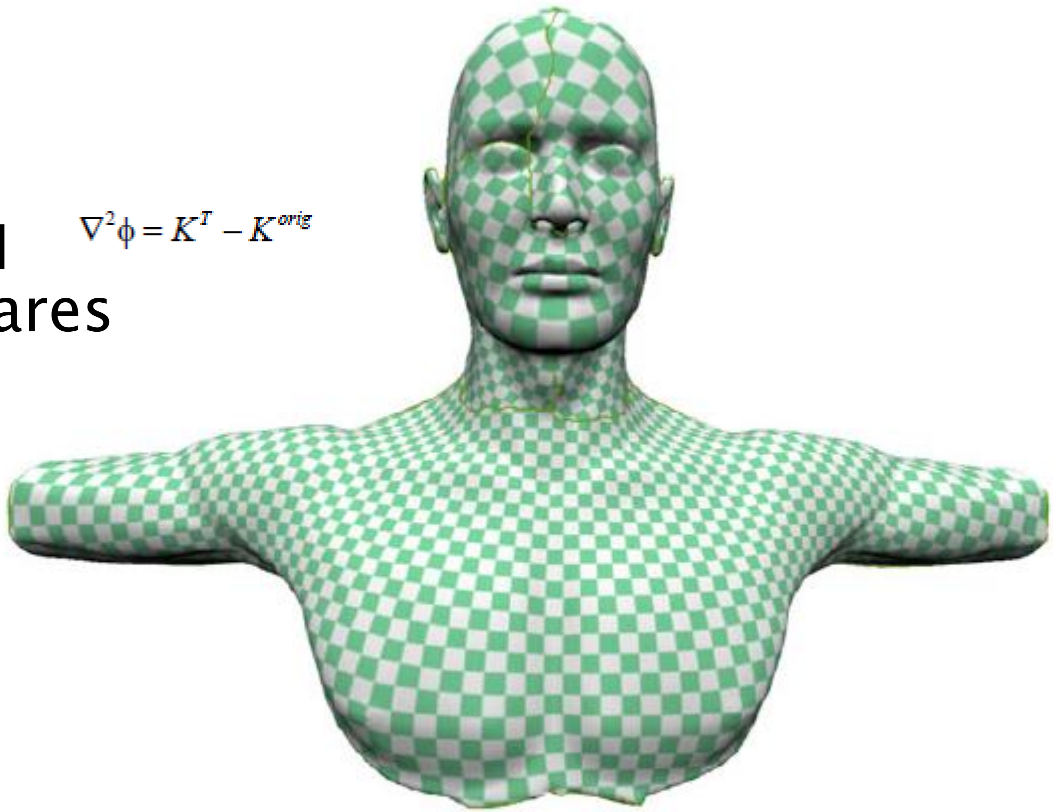
$$\sum_{v\in M, v\notin \partial M} K_v + \sum_{v\in \partial M} \kappa_v = 2\pi\chi(M)$$



[18]

Our algorithm

- Key References
  - CFCPMS
    - Poisson eq $\nabla^2\phi = K^T - K^{orig}$
    - Least-squares

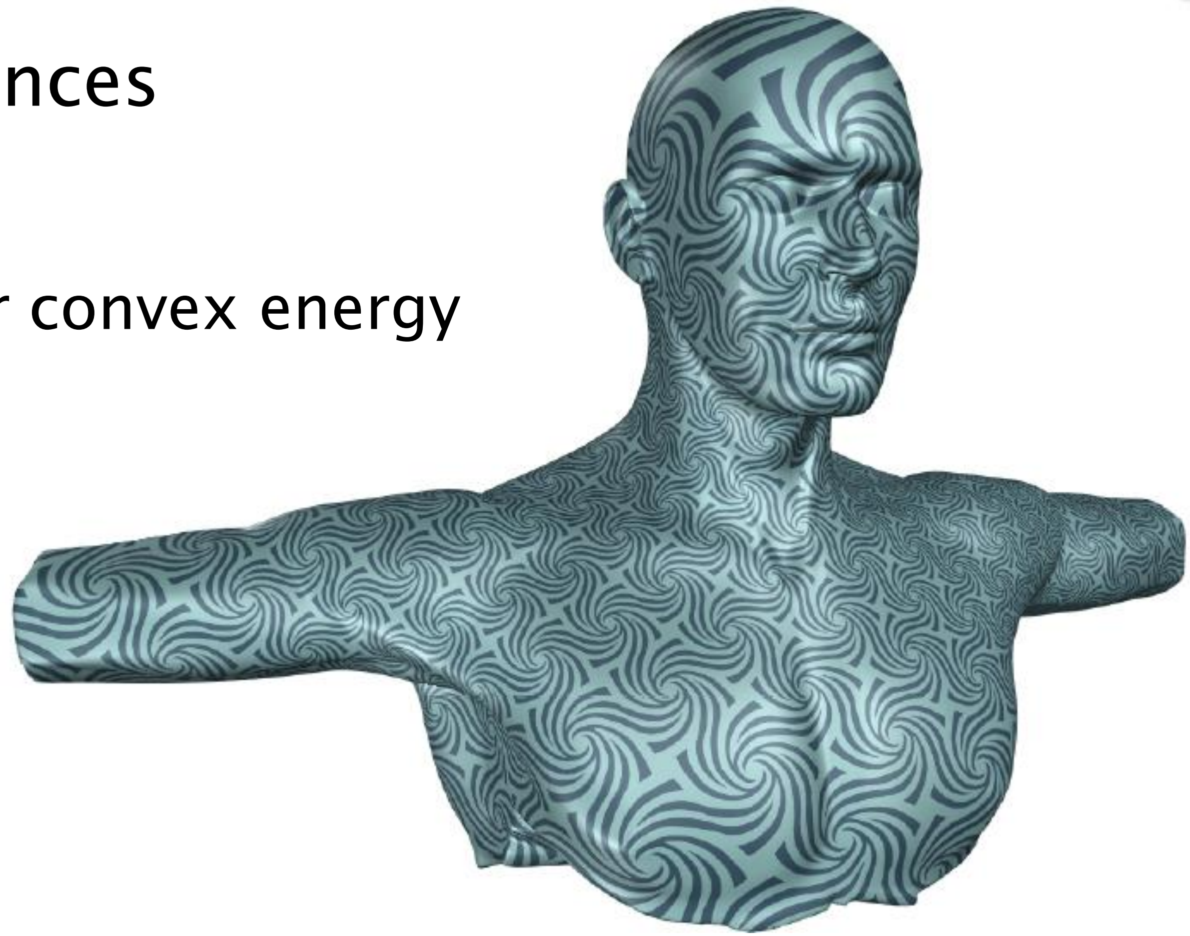


[2]

Our algorithm

- Key References
  - CETM

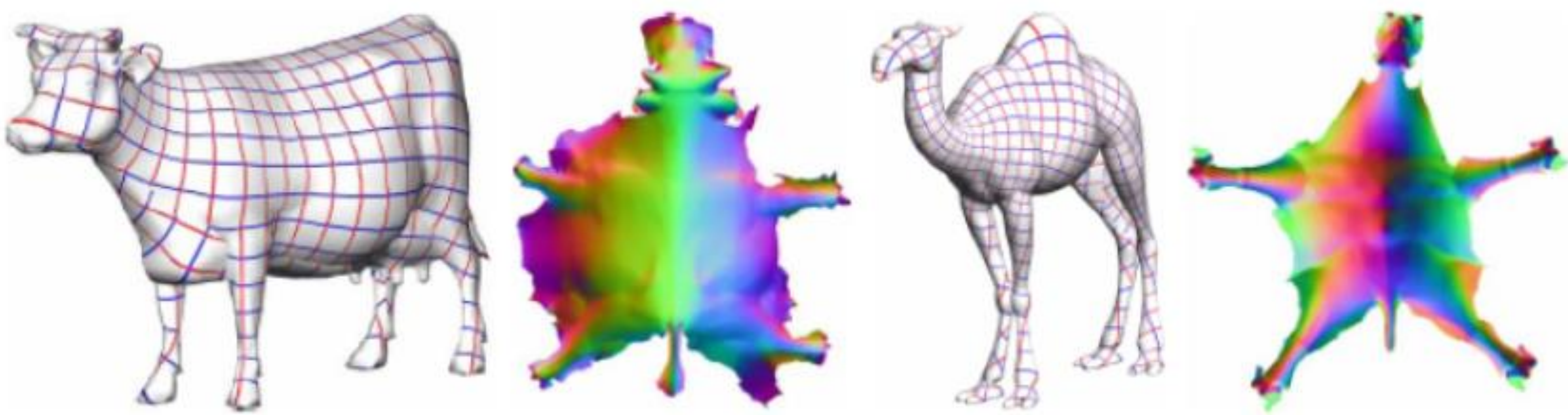
  - Non-linear convex energy



[11]

Our algorithm

- Key References
  - ABF++
    - Non-linear optimization problem
    - Slow



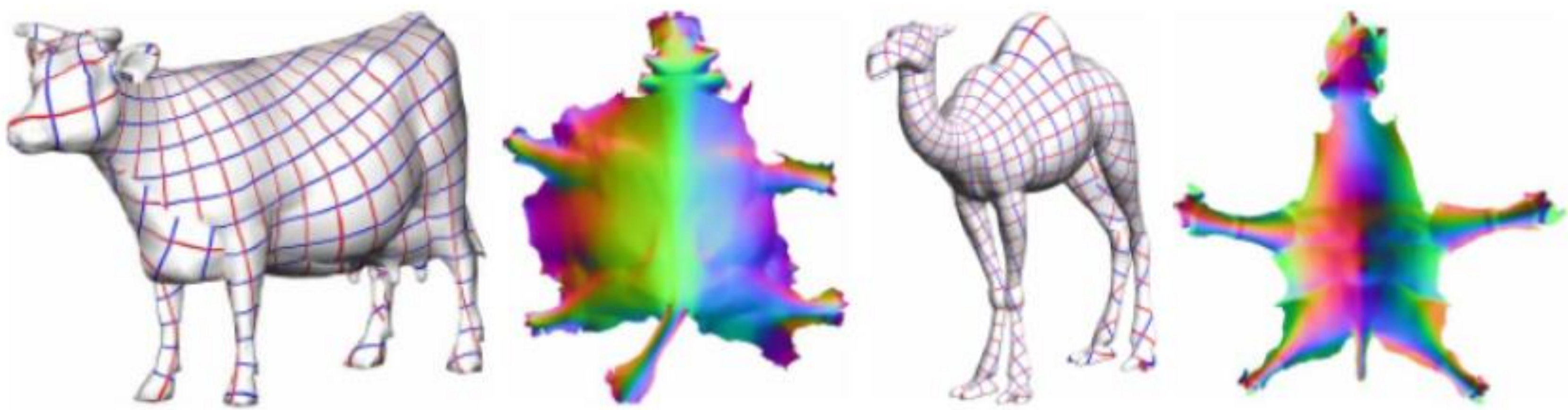
[10]

Our algorithm

- Key References
  - MIPS
    - Non-linear optimization problem
    - Slow

$$K_2(\mathbf{J}_T) = \|\mathbf{J}_T\|_2 \|\mathbf{J}_T^{-1}\|_2 = \sigma_1/\sigma_2$$

$$K_F(\mathbf{J}_T) = \|\mathbf{J}_T\|_F \|\mathbf{J}_T^{-1}\|_F = \frac{\sigma_1^2+\sigma_2^2}{\sigma_1\sigma_2} = \frac{\mathrm{trace}(\mathbf{I}_T)}{\det(\mathbf{J}_T)}$$

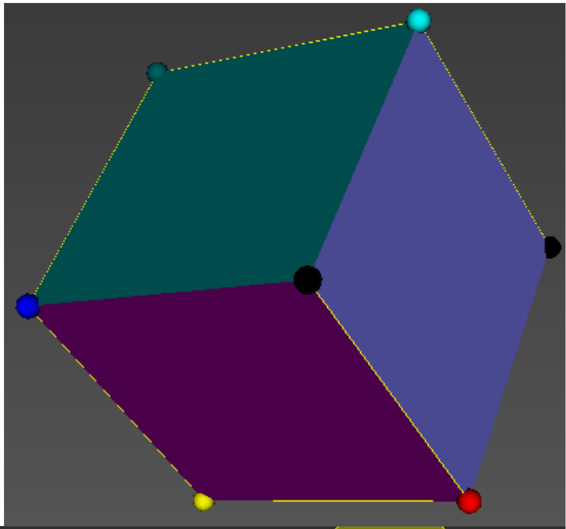


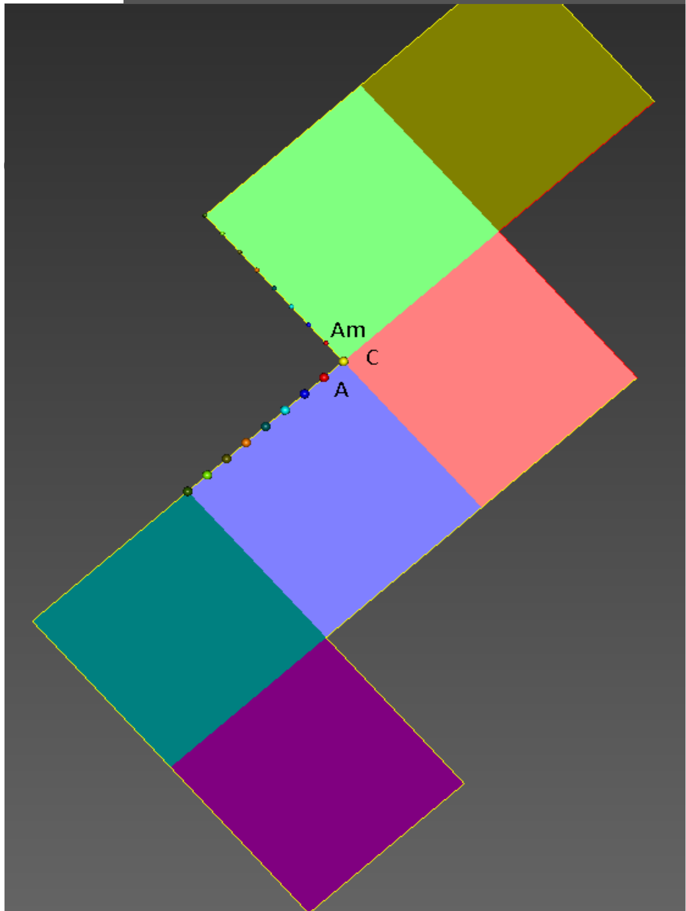
[4]

Our algorithm

- LSCM
  - Improvement:
    - Add rotational terms to the distortion energy.
    - Detect the angle of a cone singularity
    - Round it to the nearest value multiple of pi/2
    - Constrain that angle to the new value
    - Translation, rotation, translation

$$\begin{pmatrix} mA.x \\ mA.y \\ 1 \end{pmatrix} = T1 * R * T2 * \begin{pmatrix} A.x \\ A.y \\ 1 \end{pmatrix}$$

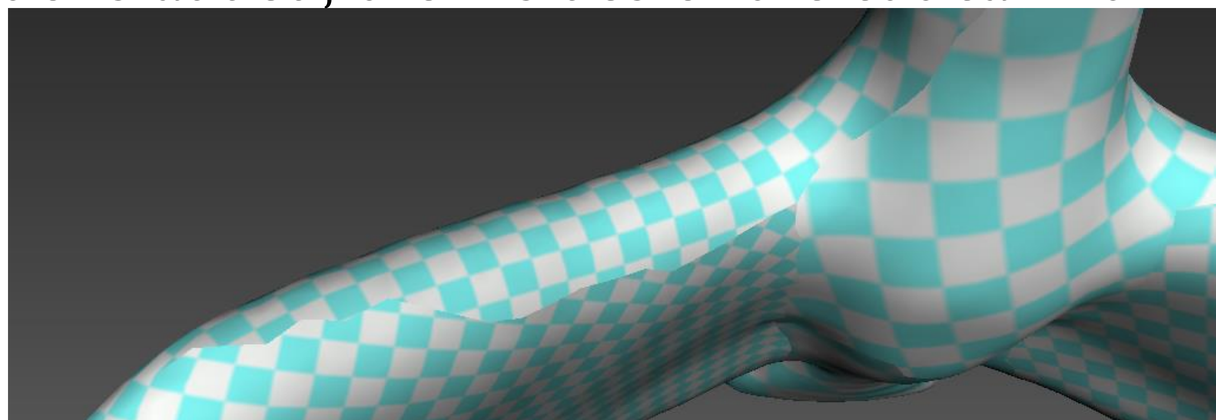




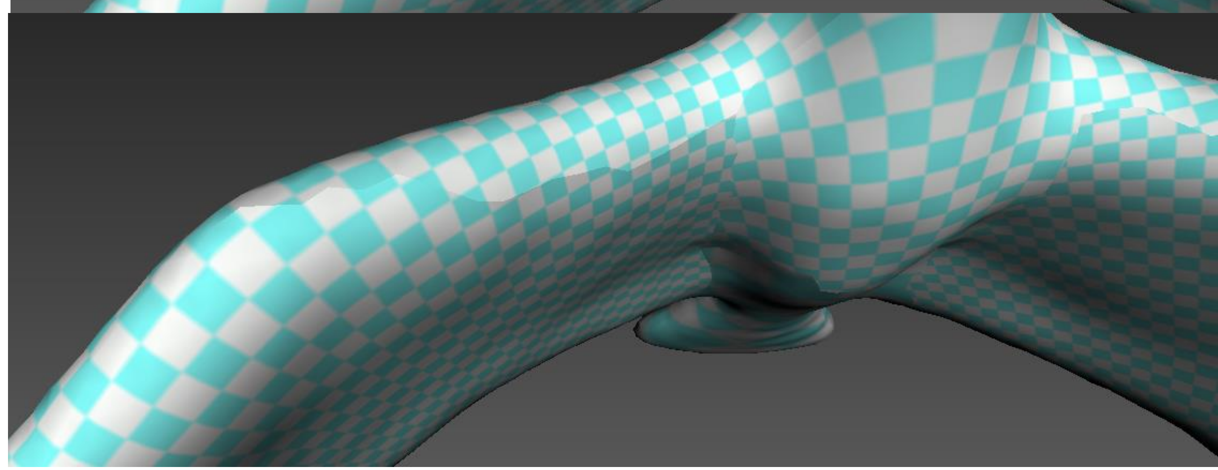
Our algorithm

- LSCM
  - With rotation equations added, the 2 sides of the cut can fit seamlessly

LSCM

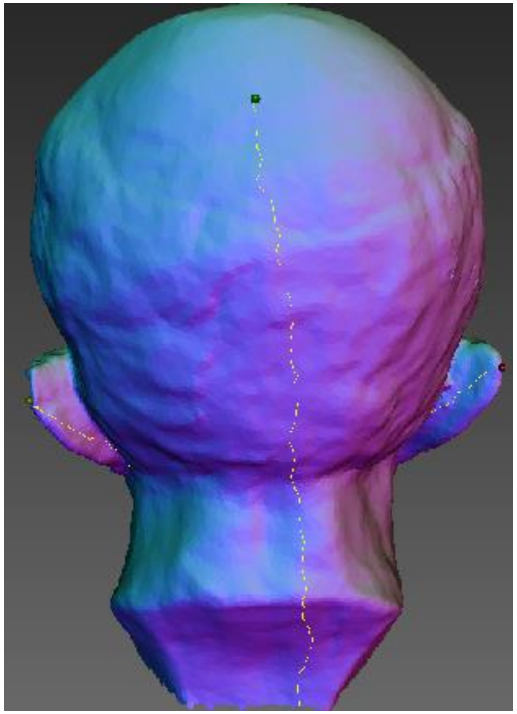
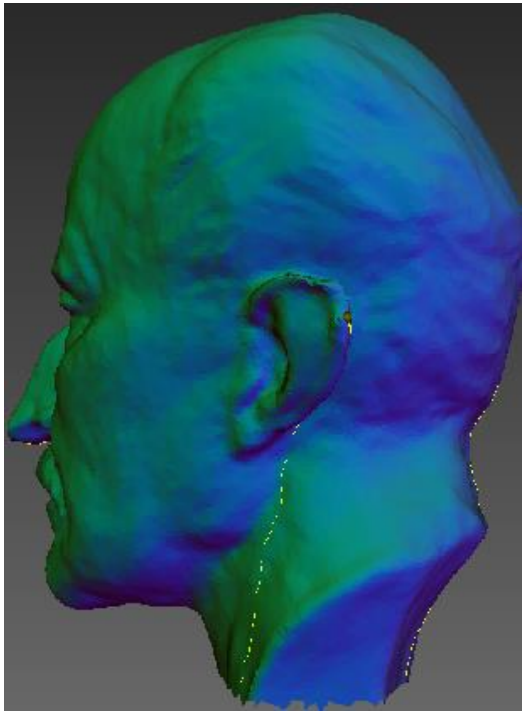
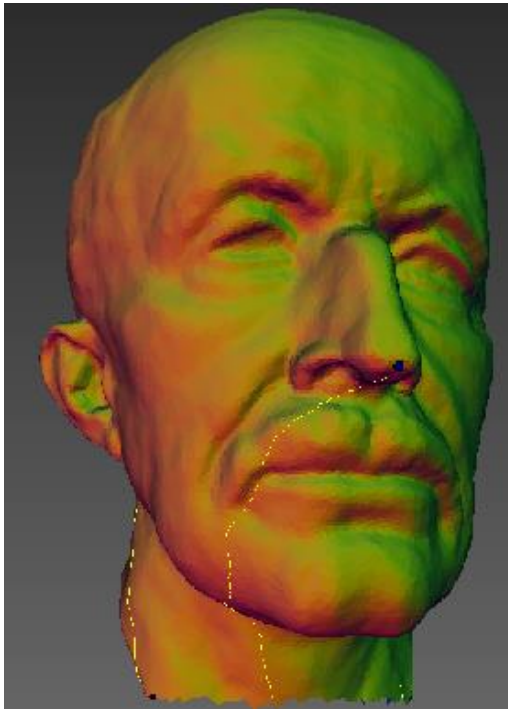


LSCM+rot



Experiments

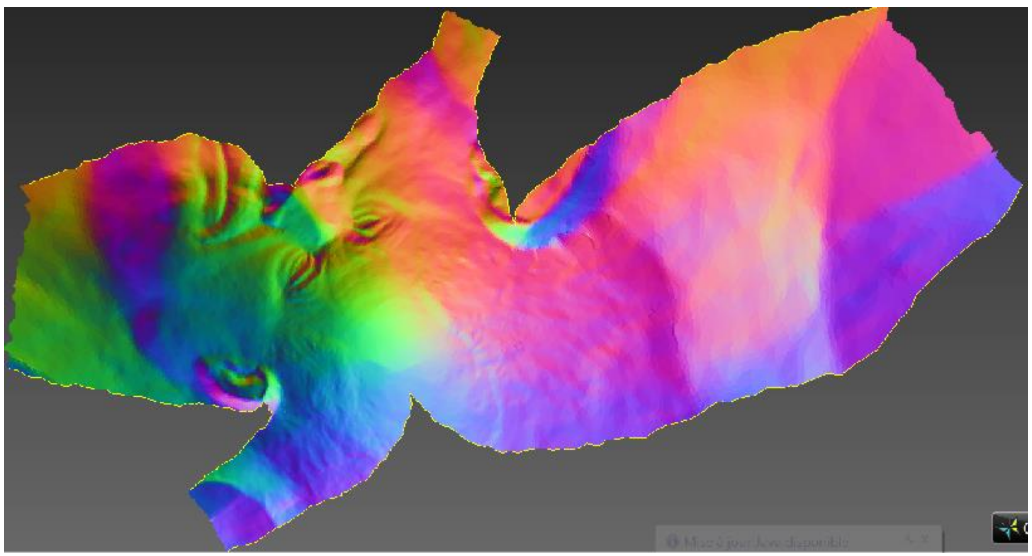
- Mesh “Planck” – 23525V, 46930F
- Manually placed cones

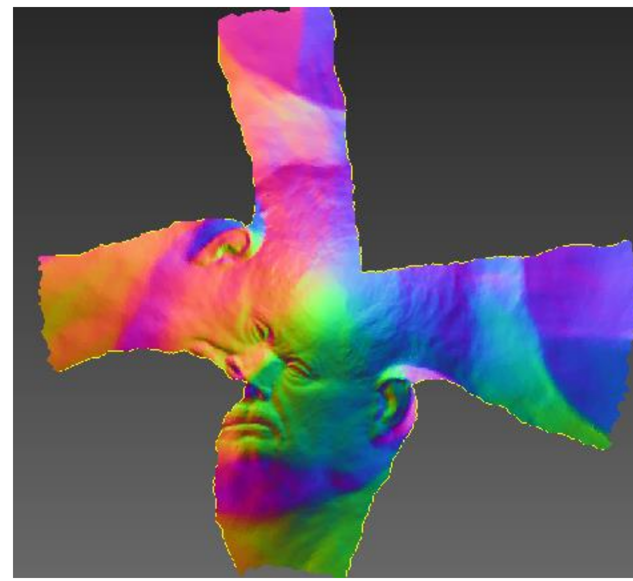
Experiments

- LSCM [2] and rotational equations
- Resulted flattening

LSCM



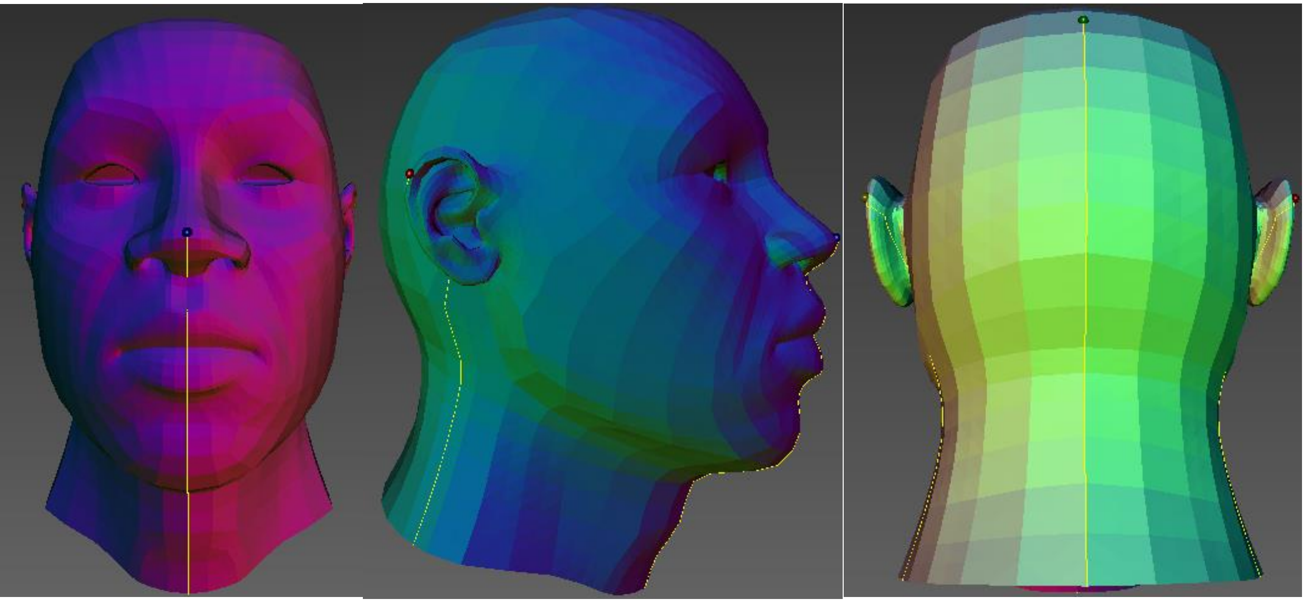
LSCM+rot



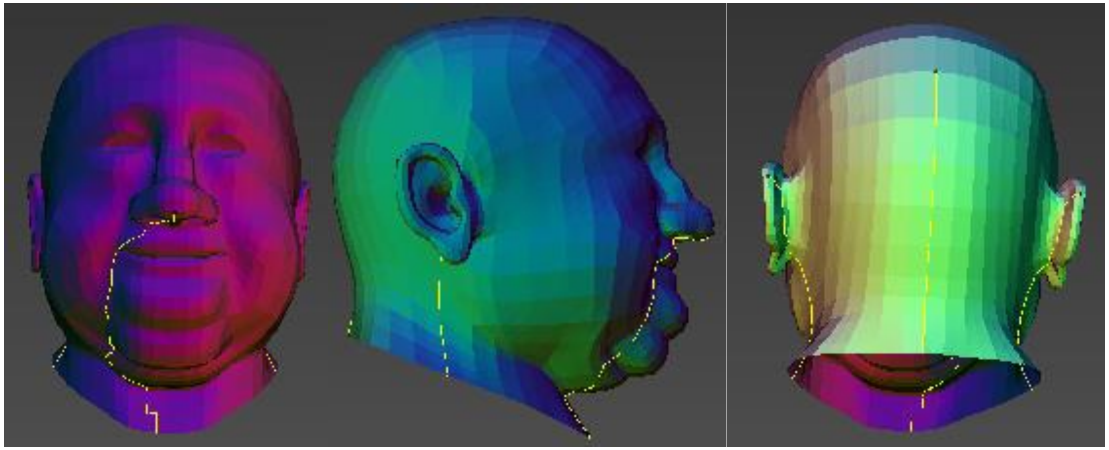
Experiments

- Try cross-map between near isometric meshes

Mesh head2q
10857V
21656F

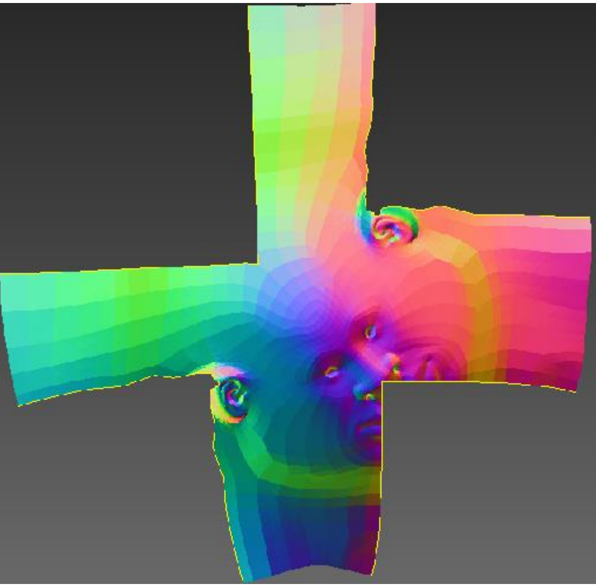


Mesh head3q
9429V
18792F

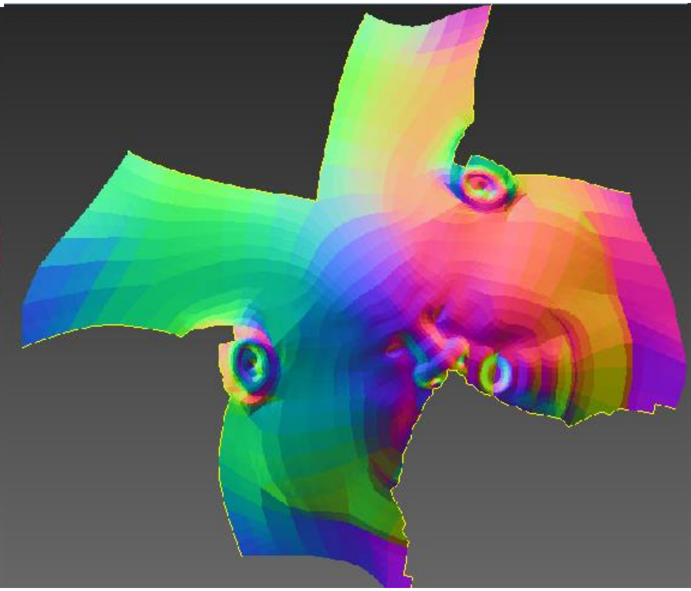


Experiments

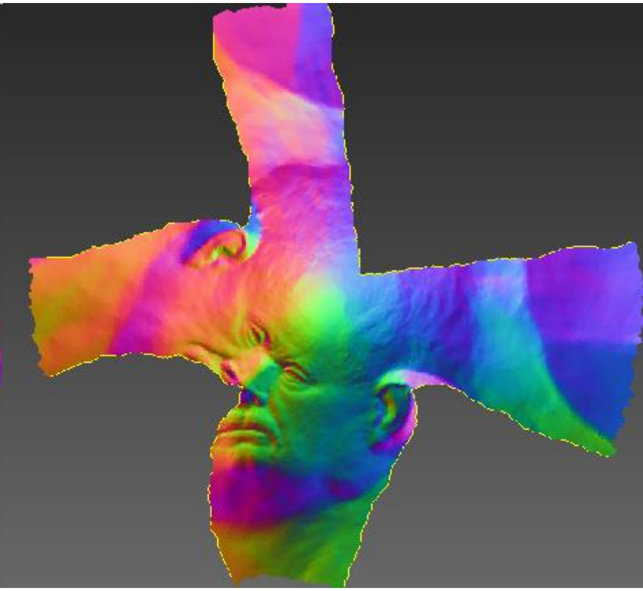
- Planck, head2q, head3q – manually placed cones
  - Visualize the meshes unfolded with the new alg (LSCM+rot)



Head2q



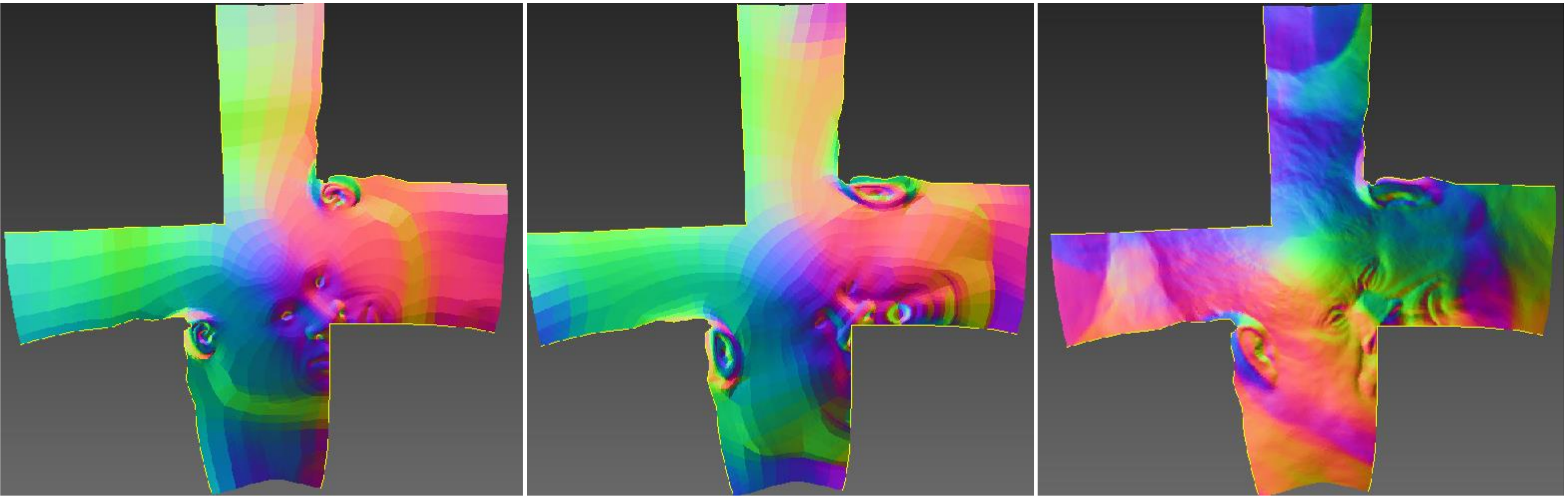
Head3q



Planck

Experiments

- Try cross-map between near isometric meshes
  - First unfold head2q with the new algorithm (LSCM+rot)
  - Pin the boundary vertices of head3q and Planck to match the boundary vertices of head2q



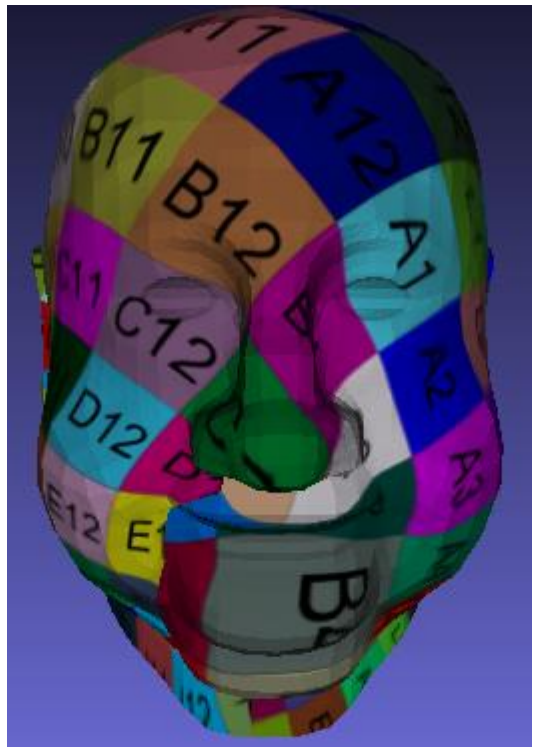
Head2q Head3q_to_2q Planck_to_2q

OBS: known cones corresp -> known corresp cut-paths

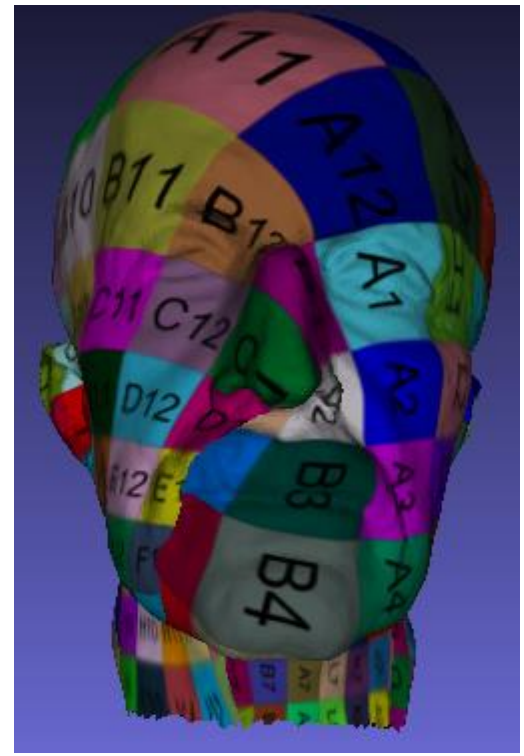
Experiments

- Same texture applied to the 3 meshes constrained to the boundary of Head2q

Head2q



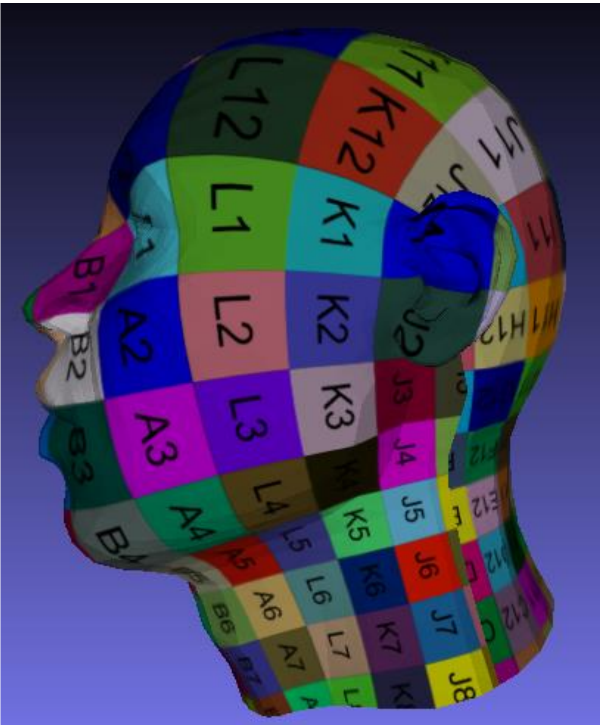
Head3q_to_2q



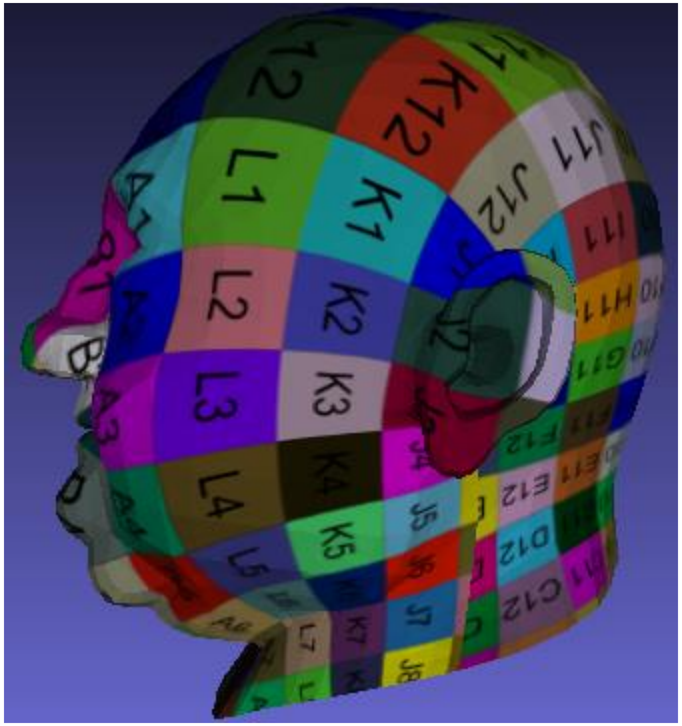
Planck_to_2q

Experiments

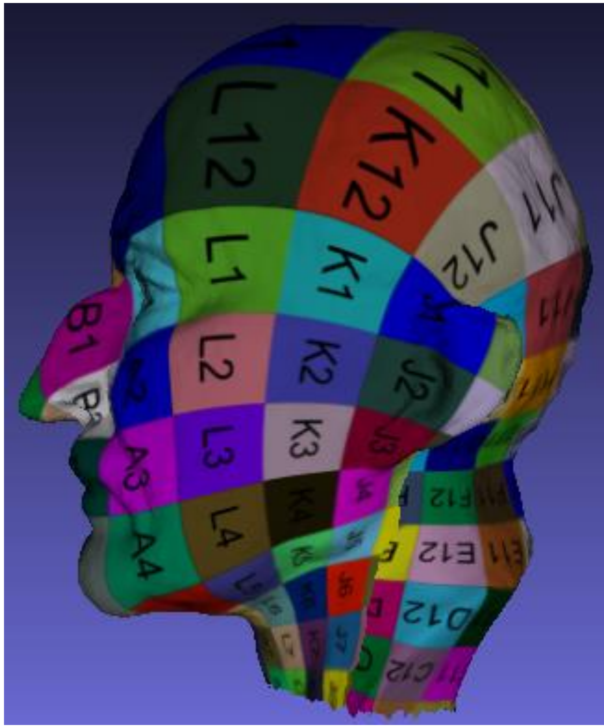
- Same texture applied to the 3 meshes constrained to the boundary of Head2q



Head2q



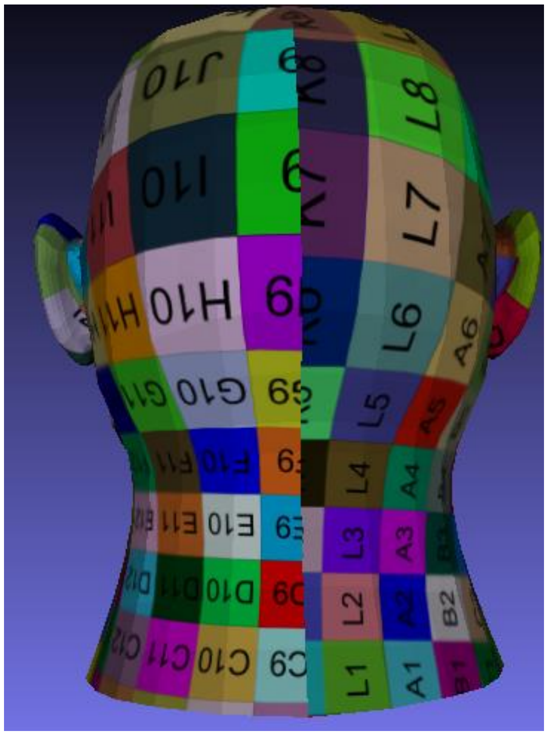
Head3q_to_2q



Planck_to_2q

Experiments

- Same texture applied to the 3 meshes constrained to the boundary of Head2q



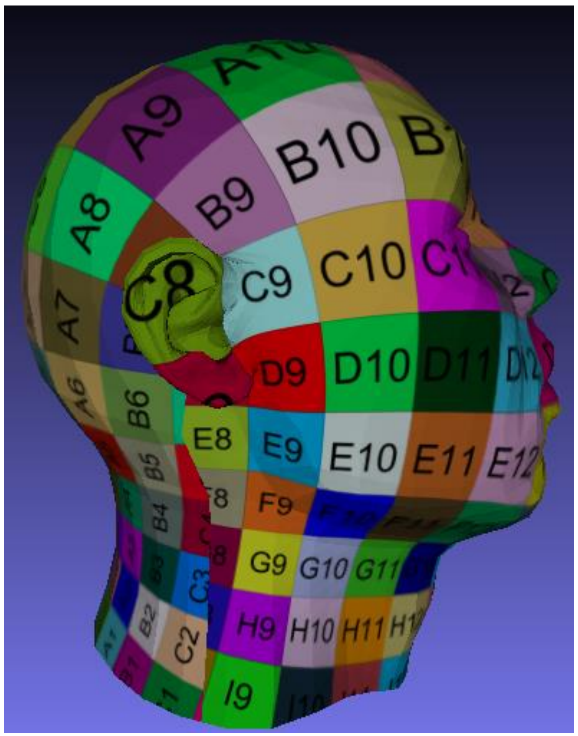
Head2q

Head3q_to_2q

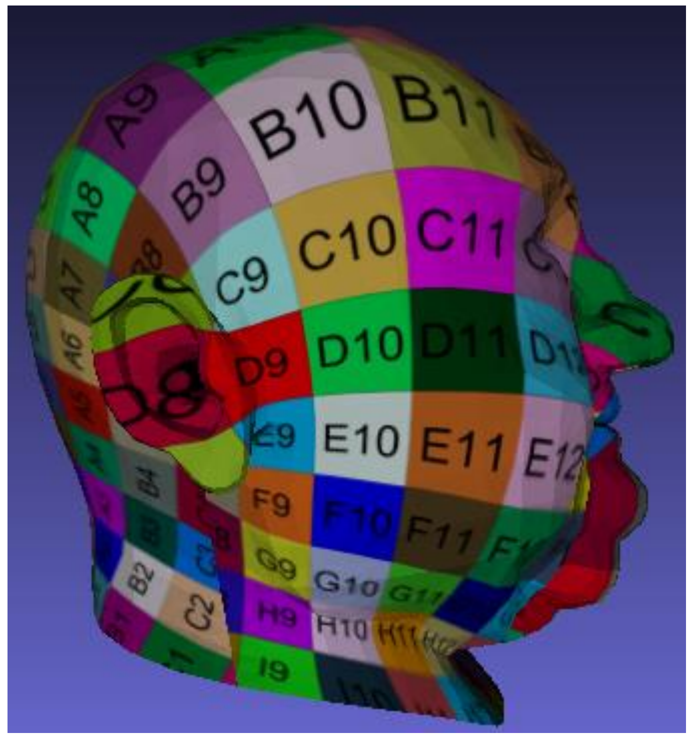
Planck_to_2q

Experiments

- Same texture applied to the 3 meshes constrained to the boundary of Head2q



Head2q

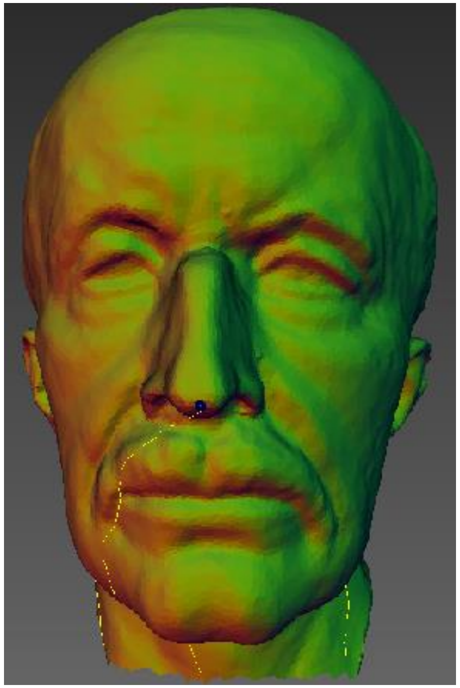


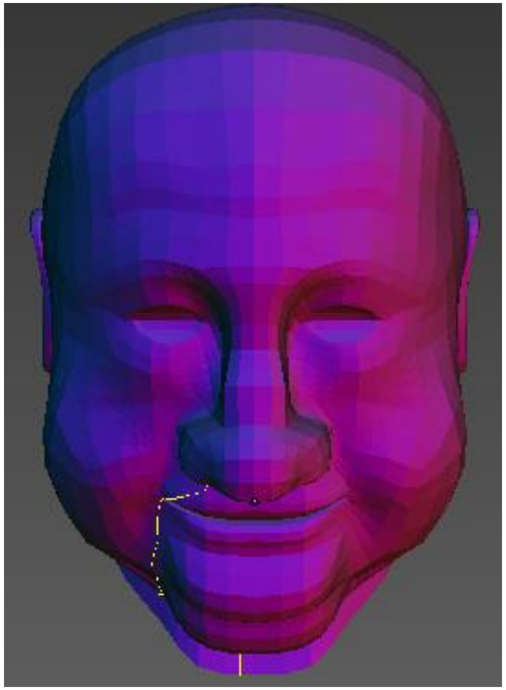
Head3q_to_2q

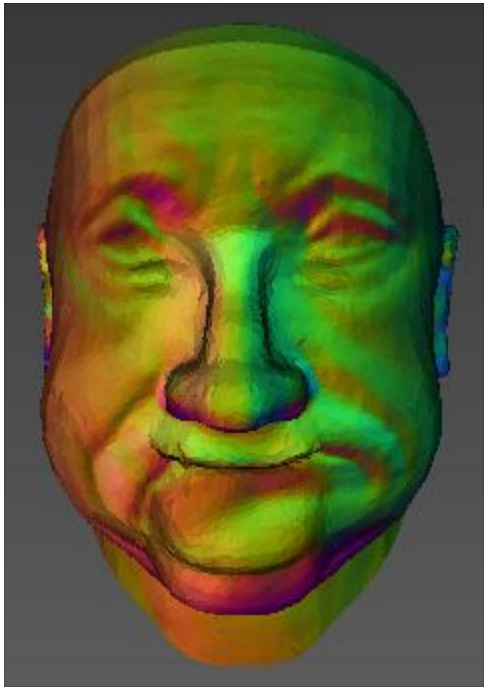
Planck_to_2q

Experiments

- Try cross-map between near isometric meshes
  - Since for both meshes head3q and Planck, the cut2 are in similar locations, do a cross-map between them
  - Map Planck to head3q, color by faces' normals

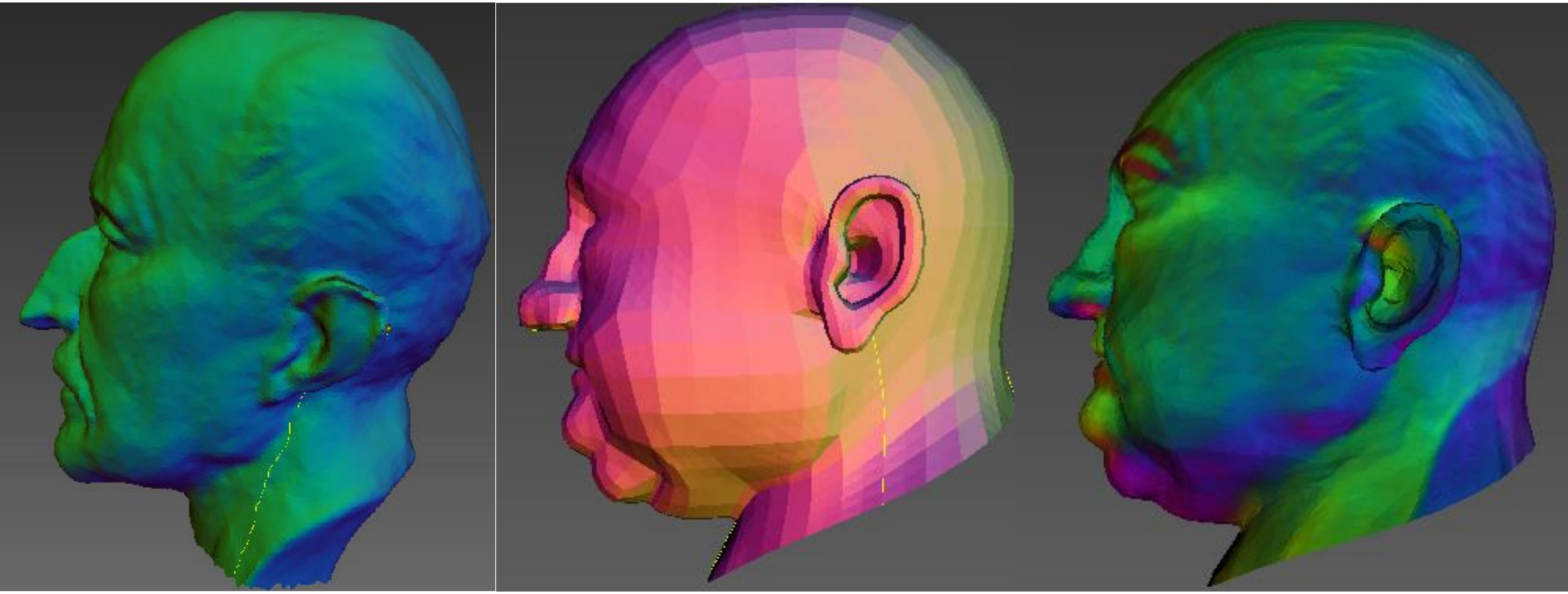






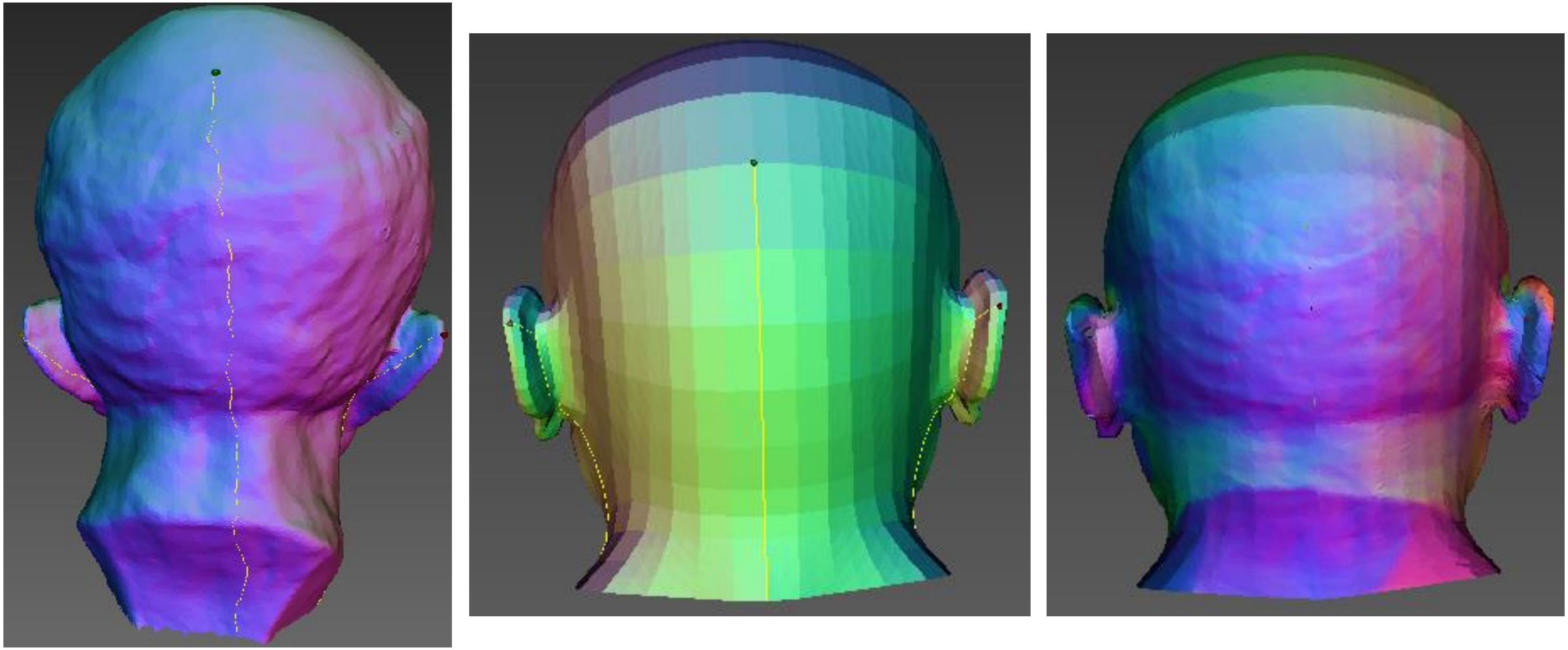
Experiments

- Try cross-map between near isometric meshes
  - Map Planck to head3q, color by faces' normals



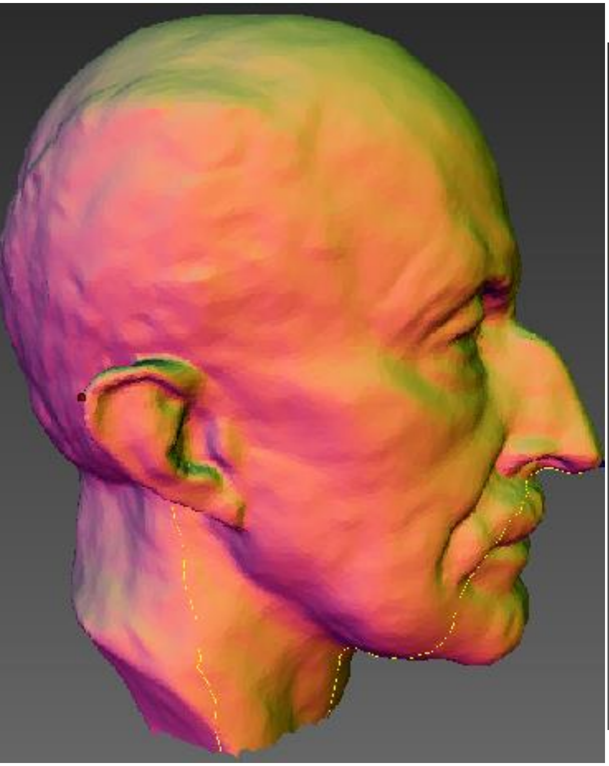
Experiments

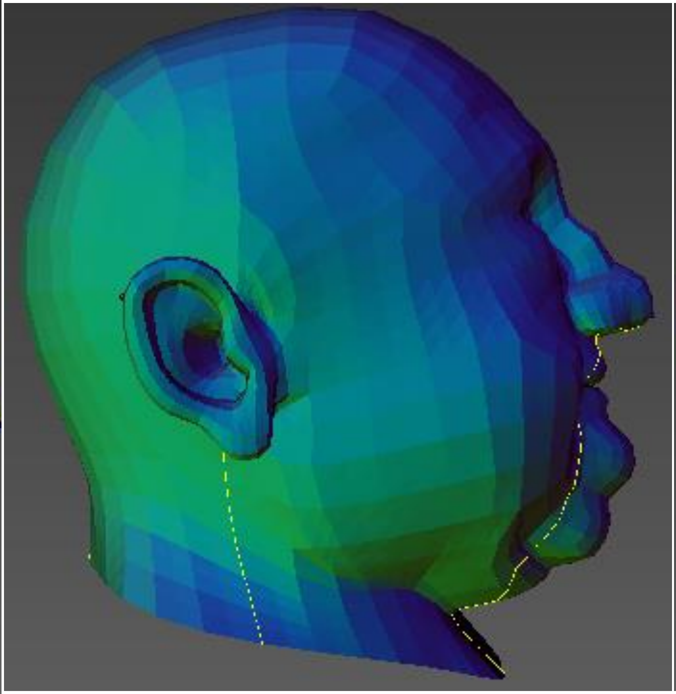
- Try cross-map between near isometric meshes
  - Map Planck to head3q, color by faces' normals

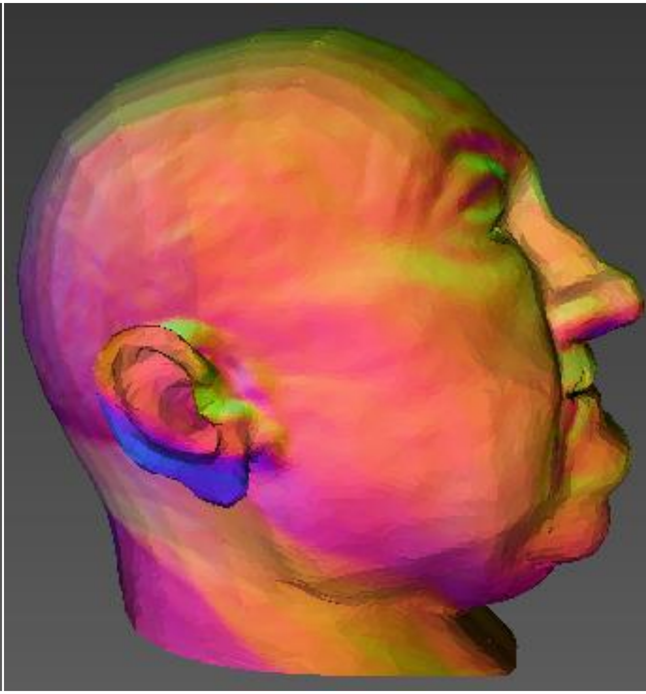


Experiments

- Try cross-map between near isometric meshes
  - Map Planck to head3q, color by faces' normals

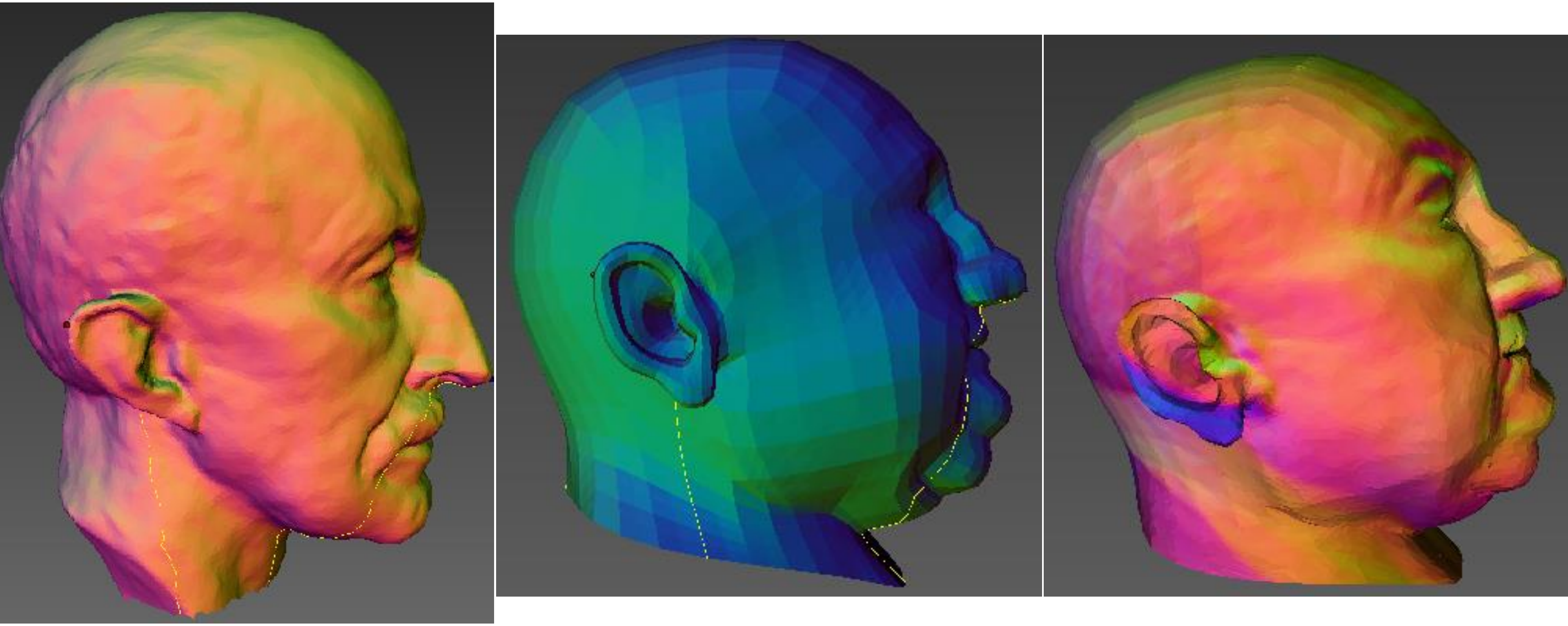






Experiments

- Try cross-map between near isometric meshes
  - Apply the same texture to all 3 flattenings; visualize 3D



Experiments

- Quasi-conformal factor
  - Ratio of the larger to the smaller eigenvalue of the Jacobian matrix -> ideal = 1

<table>
<tr><th>Map / Mesh</th><th>LSCM</th><th>LSCM+rot</th><th>LSCM+pinned bdry</th><th>Cross-map</th></tr>
<tr><td rowspan="2">Head2q</td><td rowspan="2">1.0024</td><td rowspan="2">1.0028</td><td rowspan="2">1.0028</td><td>1.3030</td></tr>
<tr><td rowspan="2">1.5268</td></tr>
<tr><td rowspan="2">Head3q</td><td rowspan="2">1.0034</td><td rowspan="2">1.0034</td><td rowspan="2">1.1002</td></tr>
<tr><td rowspan="2">1.3131</td></tr>
<tr><td rowspan="2">Planck</td><td rowspan="2">1.0002</td><td rowspan="2">1.0002</td><td rowspan="2">1.0350</td></tr>
<tr><td>1.3030</td></tr>
</table>

Experiments

- Timings [s]

Mapping / Mesh	LSCM	LSCM+rot	LSCM+pinned bdry*	Cross-map
Head2q 10857V, 21656F	1.942444	2.305175	–	1705.965937 670.079407
Head3q 9429V, 18792F	1.547186	1.833591	4.261377	1517.296428
Planck 23525V, 46930F	7.944052	9.237458	15.169150	1705.965937

* pinned bry verts to head2q flattening

Perspectives

- Initial user-driven cross-map for simple configurations
  - User-supplied corresponding cone singularities
- Good performance
- Good timings for the 2D parameterization
- Existence of solutions to speed up the cross-map

Perspectives

- More general alg to support arbitrary cut networks/ arbitrary singularity layouts
- Automatic -> pairs of corresponding cone singularities and consistent cuts on two models
- Post-process procedure for the planar optimization

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